Nov 7 Math 2253H sec. 05H Fall 2014

Section 5.1: Area Between Curves

Horizontal Orientation

The area A bounded between the continuous curves $x = f(y)$, $x = g(y)$ and the horizontal lines $y = c$ and $y = d$ is

$$A = \int_c^d |f(y) - g(y)|\, dy.$$

Note that the condition $f(y) \geq g(y)$ would mean that the curve $x = f(y)$ is to the **right** of the curve $x = g(y)$.

Find the area bounded between $x = y^2 - 1$ and $y = \frac{x-2}{2}$.

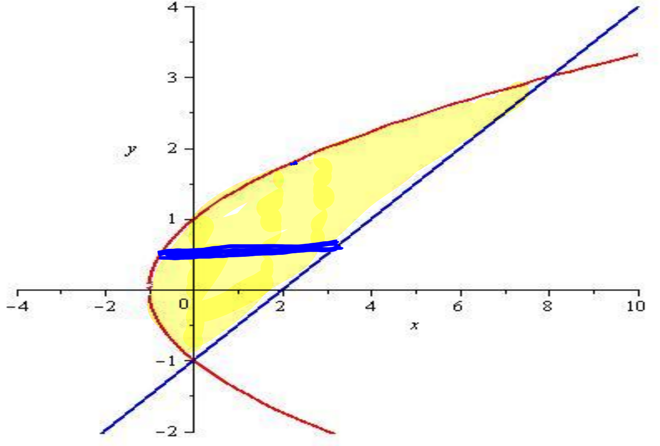

Figure: Region enclosed by $y = \frac{x-2}{2}$ and $y = \pm\sqrt{x+1}$ (a.k.a $x = y^2 - 1$).

$y = \frac{x-2}{2} \Rightarrow 2y = x - 2 \Rightarrow x = 2y + 2$

The two curves are $x = y^2 - 1$ and $x = 2y + 2$

Find the intersections: $y^2 - 1 = 2y + 2 \Rightarrow$

$y^2 - 2y - 3 = 0 \Rightarrow (y+1)(y-3) = 0$

$\Rightarrow y = -1$ or $y = 3$

Area $A = \int_{-1}^{3} \left(2y + 2 - (y^2 - 1)\right) dy$

$$= \int_{-1}^{3} (2y + 2 - y^2 + 1)\, dy$$

$$= \int_{-1}^{3} (2y + 3 - y^2)\, dy$$

$$= y^2 + 3y - \frac{y^3}{3} \Big|_{-1}^{3}$$

$$= \left(3^2 + 3 \cdot 3 - \frac{3^3}{3}\right) - \left((-1)^2 + 3(-1) - \frac{(-1)^3}{3}\right)$$

$$= 9 - \left(-2 + \frac{1}{3}\right) = 11 - \frac{1}{3} = \frac{33-1}{3} = \frac{32}{3}$$

Section 5.2: Volumes

We'll call an object a **cylinder** if cross sections taken with respect to some axis are identical.

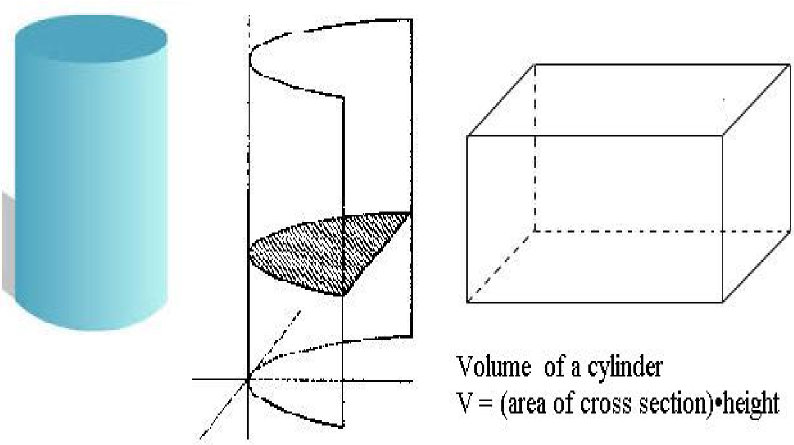

Figure: A circular, a parabolic, and a rectangular cylinder.

Volume of Solid by Slicing

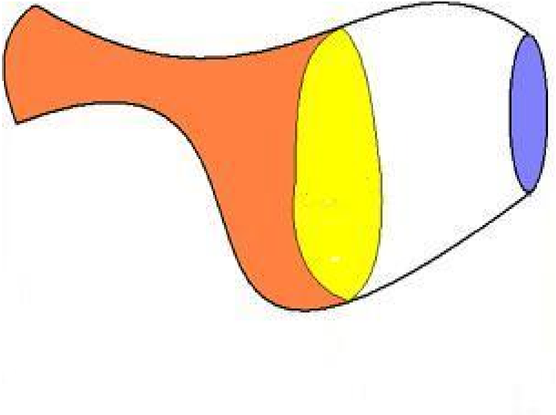

Figure: Suppose we have a solid that isn't actually a cylinder.

Volume of Solid by Slicing (Motivated by Bread)

Figure: Suppose we wish to find the volume of a loaf of bread.

Volume of Solid by Slicing (Motivated by Bread)

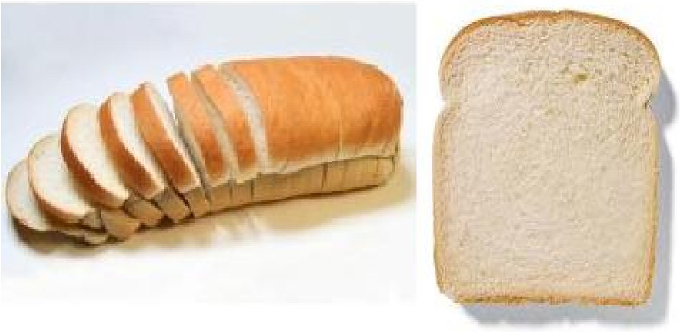

Figure: We can slice the loaf into pieces, and add the volumes of the slices.

Volume of Solid by Slicing

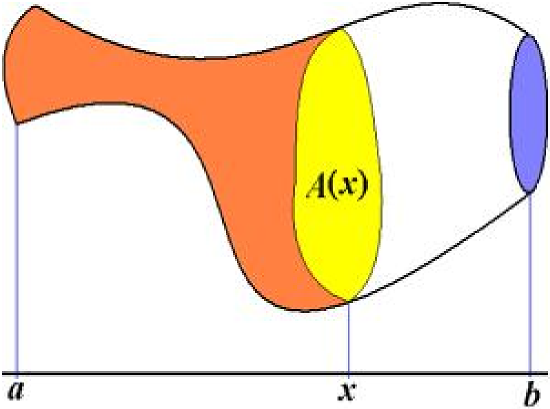

Figure: We place our solid over an x-axis for $a \leq x \leq b$. And consider a cross section at some x value.

Volume of Solid by Slicing

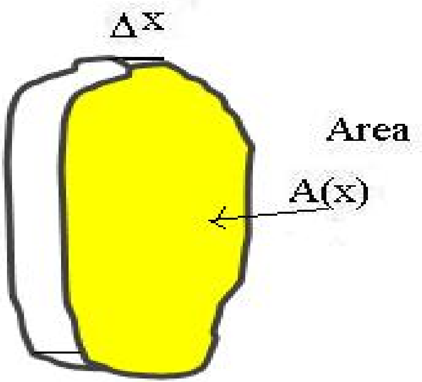

Figure: If we take slices at x and $x + \Delta x$ and remove a piece, it is *nearly* a cylinder of volume $V = A(x)\Delta x$.

Volume of Solid by Slicing

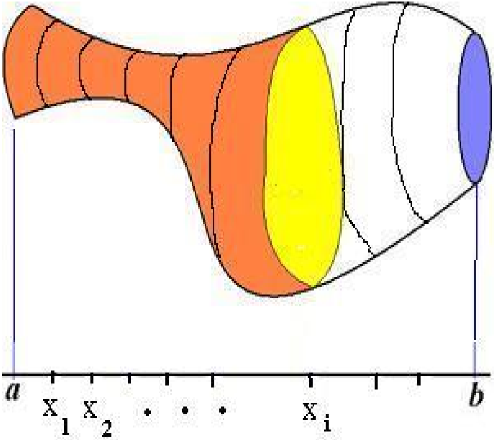

Figure: The total volume $V \approx A(x_1)\Delta x + A(x_2)\Delta x + \cdots + A(x_n)\Delta x$.

Volume of Solid by Slicing

Our volume

$$V \approx A(x_1)\Delta x + A(x_2)\Delta x + \cdots + A(x_n)\Delta x = \sum_{i=1}^{n} A(x_i)\Delta x$$

Volume: Let S be a solid that that lies between $x = a$ and $x = b$ having cross sectional area $A(x)$, where the cross section is in the plane through the solid perpendicular to the x-axis at each x in (a, b). The volume of S is

$$V = \lim_{n\to\infty} \sum_{i=1}^{n} A(x_i)\Delta x = \int_a^b A(x)\,dx.$$

An object has as its base the disk $x^2 + y^2 \leq 4$ in the xy-plane. Cross sections taken perpendicular to the x-axis are squares with one side in the plane. Find the volume of the solid.

We'll pick this up next time.

The button below takes you to a java applet illustrating this situation.

▸ Volume by Cross Section Applet