Oct 7 Math 2253H sec. 05H Fall 2014

Section 3.5: Graphing

Analyze the function $f(x) = 5x^{2/3} - 2x^{5/3}$, and use the results to produce a rough plot of the graph $y = f(x)$.

We already found the following stuff:

- The domain is all reals,
- the y-intercept is at $(0,0)$, and the x-intercepts are $(0,0)$ and $(5/2, 0)$
- f has no vertical or horizontal asymptotes, and $f(x) \to -\infty$ as $x \to \infty$ while $f(x) \to \infty$ as $x \to -\infty$
- f has first and second derivatives

$$f'(x) = \frac{10(1-x)}{3x^{1/3}}, \quad \text{and} \quad f''(x) = \frac{-10(1+2x)}{9x^{4/3}}$$

Analyze $f'(x)$: Critical #:

$f'(x) = 0 \implies 10(1-x) = 0 \implies x = 1$

$f'(x)$ DNE $\implies 3x^{1/3} = 0 \implies x = 0$

Crit # are 1 and 0

Sign of f'

$$\underset{0}{\overset{-}{\longleftarrow}}\ \Big|\ \overset{+}{\longrightarrow}\ \underset{1}{\Big|}\ \overset{-}{\longrightarrow}$$

$f'(-1)$ $\frac{(+)}{(-)}$ $\qquad f'(\frac{1}{2})$ $\frac{(+)}{(+)}$ $\qquad f'(2)$ $\frac{(-)}{(+)}$

f is increasing on $(0,1)$ and decreasing on $(-\infty,0) \cup (1,\infty)$.

By the 1^{st} derivative test f has a local min

@ $(0,0)$.

$$f(1) = 5(1)^{2/3} - 2(1)^{5/3} = 5-2 = 3$$

By the 1^{st} deri. test f has a local max

@ $(1,3)$.

2nd Derivative Analysis: $f''(x) = \dfrac{-10(1+2x)}{9x^{4/3}}$

$$f''(x) = 0 \Rightarrow -10(1+2x) = 0 \Rightarrow x = \frac{-1}{2}$$

$$f''(x) \text{ DNE} \Rightarrow 9x^{4/3} = 0 \Rightarrow x = 0$$

Sign of f''

$$\begin{array}{ccccc} & + & & - & - \\ \longleftarrow & & -1/2 & & 0 & \longrightarrow \end{array}$$

$f''(-1)\ \frac{(+)}{(+)}$ $\quad$ $f''\left(\frac{-1}{4}\right)\ \frac{(-)}{(+)}$ $\quad$ $f''(1)\ \frac{(-)}{(+)}$

f is concave up on $(-\infty, \frac{-1}{2})$ and concave down on $(-\frac{1}{2}, 0) \cup (0, \infty)$

$$f\left(\frac{-1}{2}\right) = 5\left(\frac{-1}{2}\right)^{2/3} - 2\left(\frac{-1}{2}\right)^{5/3} = \left(\frac{-1}{2}\right)^{2/3}\left(5 - 2\left(\frac{-1}{2}\right)\right) = 6\left(\frac{-1}{2}\right)^{2/3}$$

$$= \frac{6}{\sqrt[3]{4}}$$

f has an inflection point @ $\left(\frac{-1}{2}, \frac{6}{\sqrt[3]{4}}\right)$

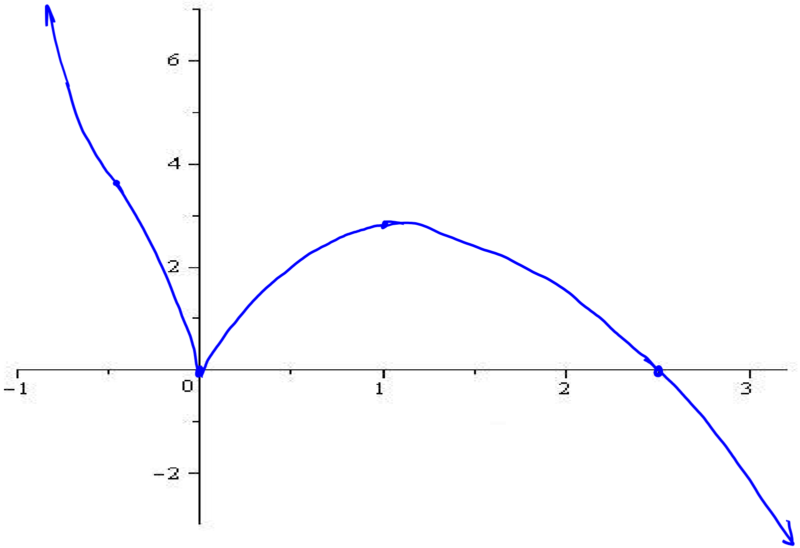


Figure: Plot produced by our analysis of $y = 5x^{2/3} - 2x^{5/3}$.

Section 3.7: Applied Optimization

Optimization problems arise in every field of study and every industry.

- minimize cost and maximize revenue,
- maximize crop yield,
- minimize driving time,
- maximize volume,
- minimize energy

Often, some constraint (extra condition) must simultaneously be satisfied.

Applied Optimization Example

A 216 m^2 rectangular pea patch is to be enclosed by a fence and divided into two equal parts by another fence parallel to one of its sides. What dimensions of the outer rectangle will require the smallest total length of fencing and how much fencing will be needed?

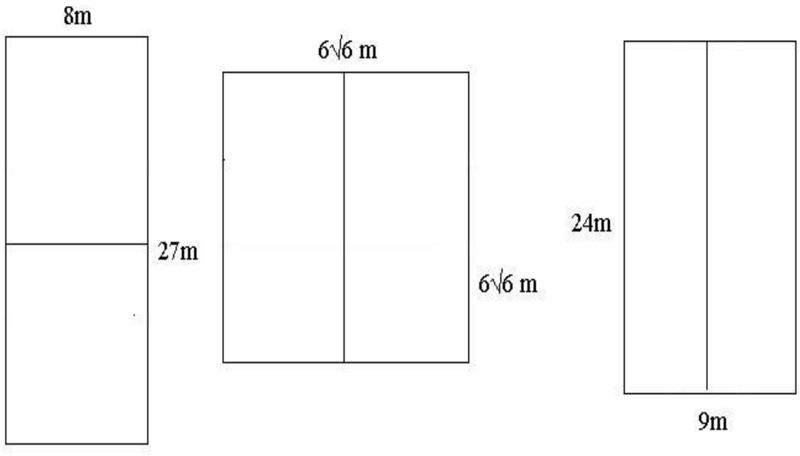
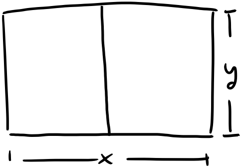


Figure: Different pea patch configuration that all enclose $216m^2$.

Consider a rectangular patch split into two equal pieces:



Two characteristic dimensions

length x

\+ width y

x, y in m

$x > 0$, $y > 0$

Given, the area enclosed $A = 216\ m^2$

so $xy = 216\ m^2$

The total amount of fencing is

objective function $\longrightarrow$ $F = 2x + 3y$ m

Task: minimize F subject to $xy = 216$.

From $xy = 216 \Rightarrow x = \frac{216}{y}$

So as a function of y alone

$$F = 2\left(\frac{216}{y}\right) + 3y$$

i.e.

$$F(y) = \frac{432}{y} + 3y, \quad y > 0$$

We want to find the y-value that minimizes F and the minimum value of F.

We'll finish this on Thursday.