Sept 12 Math 2253H sec. 05H Fall 2014

Section 2.6: Implicit differentiation

Recall Explicit -vs- Implicit Functions: A function is defined **explicitly** when given in the form

$$y = f(x).$$

A function is defined **implicitly** when it is given as a relation

$$F(x, y) = C,$$

for constant C.

Finding a Derivative Using Implicit Differentiation:

- Take the derivative of both sides of an equation with respect to the independent variable.
- Use all necessary rules for differenting powers, products, quotients, trig functions, compositions, etc.
- Remember the chain rule for each term involving the dependent variable (e.g. mult. by $\frac{dy}{dx}$ as required).
- Use necessary algebra to isolate the desired derivative.

Example

Find $\frac{dS}{dr}$.

$$\sqrt{Sr} + S = r^2 + 2$$

$$\frac{dS}{dr} = \frac{4r\sqrt{Sr} - S}{r + 2\sqrt{Sr}}$$

Example

Find the equation of the line tangent to the graph of $x^3 + y^3 = 6xy$ at the point $(3,3)$.

Let's verify that $(3,3)$ is on the curve.

left: $(3)^3 + (3)^3 = 27 + 27 = 54$

right: $6(3)(3) = 6 \cdot 9 = 54$ ✓

We need the slope m. $m = \left.\frac{dy}{dx}\right|_{(3,3)}$

Find $\frac{dy}{dx}$: $3x^2 + 3y^2\frac{dy}{dx} = 6\left(1y + x\frac{dy}{dx}\right)$

$$3y^2 \frac{dy}{dx} - 6x \frac{dy}{dx} = 6y - 3x^2$$

$$(3y^2 - 6x) \frac{dy}{dx} = 6y - 3x^2$$

$$\frac{dy}{dx} = \frac{6y - 3x^2}{3y^2 - 6x} = \frac{3(2y - x^2)}{3(y^2 - 2x)} = \frac{2y - x^2}{y^2 - 2x}$$

$$m = \frac{dy}{dx}\Big|_{(3,3)} = \frac{2(3) - 3^2}{3^2 - 2(3)} = \frac{-3}{3} = -1$$

The line is $y - 3 = -1(x - 3)$

$$\rightarrow \quad y = -x + 6$$

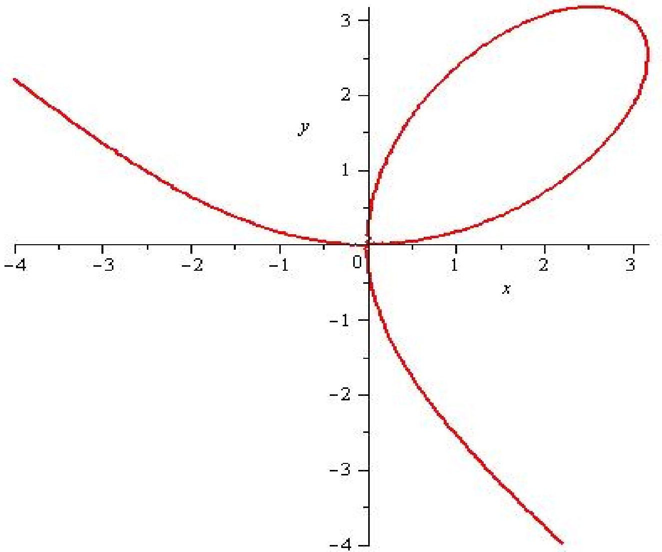

Figure: Folium of Descartes $x^3 + y^3 = 6xy$

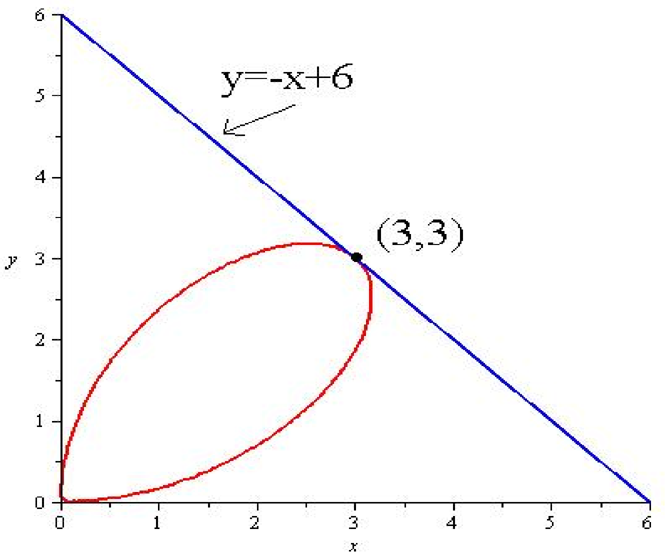

Figure: Folium of Descartes with tangent line at $(3, 3)$

Recap of the Chain Rule

Suppose t is an independent variable. If u is a function of t, and y is a function of u, then y is in turn a function of t.

When these functions are differentiable

$$\frac{dy}{dt} = \frac{dy}{du}\frac{du}{dt}.$$