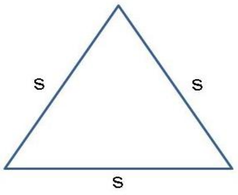
Sept 18 Math 2253H sec. 05H Fall 2014

Section 2.8 Related Rates

The sides of an equilateral triangle are increasing at a constant rate of 2 cm/min.



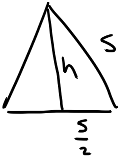
equilateral triangle

Determine the rate at which the area is increasing when
(a) the sides are 6 cm long
(b) the area is $8\sqrt{3}$ cm^2

$$A = \frac{1}{2} s h = \frac{1}{2} s \frac{\sqrt{3}}{2} s$$

$$h^2 = s^2 - \frac{s^2}{4} \Longrightarrow$$

$$h = \frac{\sqrt{3}}{2} s$$



$$\Downarrow$$

$$A = \frac{\sqrt{3}}{4} s^2$$

So $$\frac{dA}{dt} = \frac{2\sqrt{3}}{4} s \frac{ds}{dt} \Longrightarrow \boxed{\frac{dA}{dt} = \frac{\sqrt{3}}{2} s \frac{ds}{dt}}$$

a) when $s = 6$ cm

$$\frac{dA}{dt} = \frac{\sqrt{3}}{2} (6\text{ cm})\left(2 \frac{\text{cm}}{\text{min}}\right)$$

$$\frac{dA}{dt} = 6\sqrt{3} \ \frac{\text{cm}^2}{\text{min}}$$

b) when $A = 8\sqrt{3}$

$$s^2 = 8\sqrt{3} \cdot \frac{4}{\sqrt{3}} = 32$$

$$s = \sqrt{32}$$

So $$\frac{dA}{dt} = \sqrt{32}\sqrt{3} \ \frac{\text{cm}^2}{\text{min}}$$

Section 2.9: Using the tangent line (Linearization)

Suppose we have a function $y = f(x)$ whose graph passes through the point $(a, f(a))$ and that is differentiable at $x = a$.

Consider the following line

$$L(x) = f(a) + f'(a)(x - a)$$

Note $L(x) = mx + b$ where $m = f'(a)$ and $b = f(a) - af'(a)$.

Note $L(x) = \underbrace{f'(a)}_{m}\, x + \underbrace{\left(f(a) - af(a)\right)}_{b}$

$L(x) = f(a) + f'(a)(x - a)$

Example: Find $L(x)$ for $f(x) = \sqrt{x}$ and $a = 9$.

$f(x) = x^{1/2}$ $\quad$ $f'(x) = \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}}$

$f(9) = \sqrt{9} = 3$ $\quad$ $f'(9) = \frac{1}{2\sqrt{9}} = \frac{1}{2 \cdot 3} = \frac{1}{6}$

$L(x) = 3 + \frac{1}{6}(x - 9)$

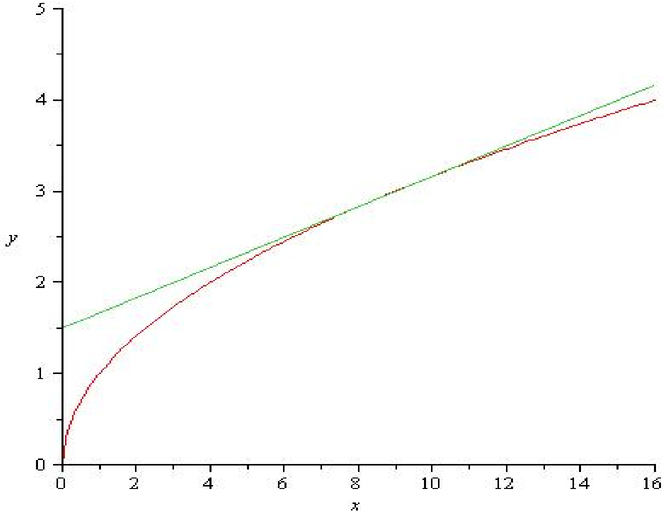


Figure: $x \approx 9$, $L(x) \approx f(x)$

Example Continued...

Show that $L(9) = f(9)$ and that $L'(9) = f'(9)$.

$L(9) = 3 + \frac{1}{6}(9 - 9) = 3$ $\quad$ $f(9) = 3$

$L'(x) = \frac{1}{6} \Rightarrow L'(9) = \frac{1}{6}$ and $f'(9) = \frac{1}{6}$

Note: $L(9.1) = 3 + \frac{1}{6}(9.1 - 9) = 3 + \frac{1}{6}\left(\frac{1}{10}\right) = \frac{180}{60} + \frac{1}{60} = \frac{181}{60}$.

$\sqrt{9.1} = 3.0166206$ (by TI-89) $\quad L(9.1) = 3.0166666$ (by hand)

5 accurate digits!

Application Example

Example: Approximate $\sqrt[4]{17}$ using a line.