Sept 19 Math 2253H sec. 05H Fall 2014

Section 2.9: Using the tangent line (Linearization)

Example: Approximate $\sqrt[4]{17}$ using a line.

We need a function f and a number a.

Let $f(x) = \sqrt[4]{x} = x^{1/4}$ and $a = 16$

So 16 is "close" to 17 and it's "nice" as a perfect 4^{th} power.

$$L(x) = f(a) + f'(a)(x-a)$$

$$f'(x) = \frac{1}{4} x^{-3/4} \implies f'(x) = \frac{1}{4(\sqrt[4]{x})^3}$$

$$f(16) = \sqrt[4]{16} = 2 \qquad f'(16) = \frac{1}{4(\sqrt[4]{16})^3} = \frac{1}{4 \cdot 2^3} = \frac{1}{32}$$

$$L(x) = 2 + \frac{1}{32}(x - 16)$$

for $x \approx 16$ $\quad L(x) \approx \sqrt[4]{x}$

$$\sqrt[4]{17} \approx L(17) = 2 + \frac{1}{32}(17 - 16) = 2 + \frac{1}{32} = \frac{64}{32} + \frac{1}{32}$$

$$= \frac{65}{32}$$

TI-89 $\sqrt[4]{17} \approx 2.0305$

$$\frac{65}{32} = 2.03125$$

Chapter 3: Applications of Differentiation

Section 3.1: Maximums and Minimums

Definition: Let f be a function with domain D and let c be a number in D. Then $f(c)$ is

- **the absolute minimum** value of f on D if $f(c) \leq f(x)$ for all x in D,
- **the absolute maximum** value of f on D if $f(c) \geq f(x)$ for all x in D.

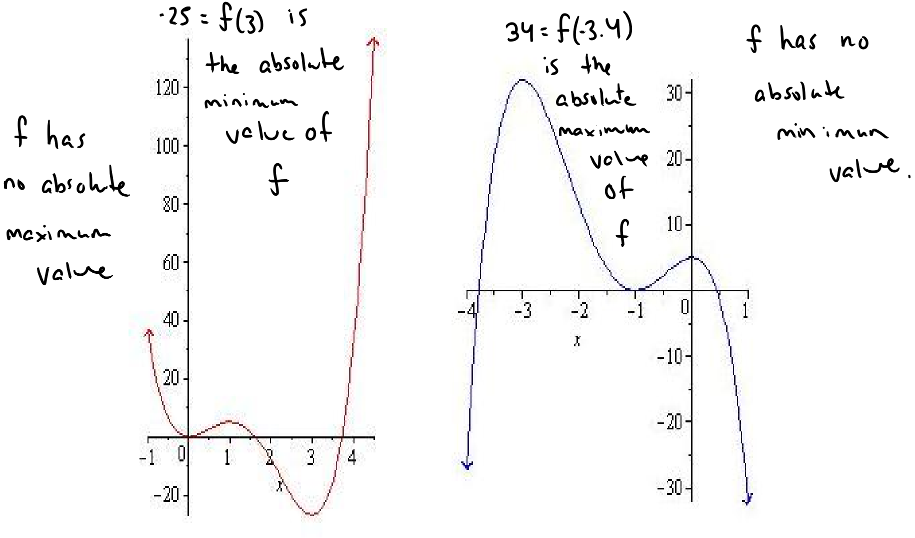


Figure: Graphically, an absolute minimum is the lowest point and an absolute maximum is the highest point.

Definition:

Let f be a function with domain D and let c be a number in D. Then $f(c)$ is

- **a local minimum** value of f if $f(c) \leq f(x)$ for x *near* c
- **a local maximum** value of f if $f(c) \geq f(x)$ for x *near* c.

more precisely

"x near c" there exists a number $\delta > 0$ such that $f(c) \leq f(x)$ for x such that

$|x - c| < \delta$.

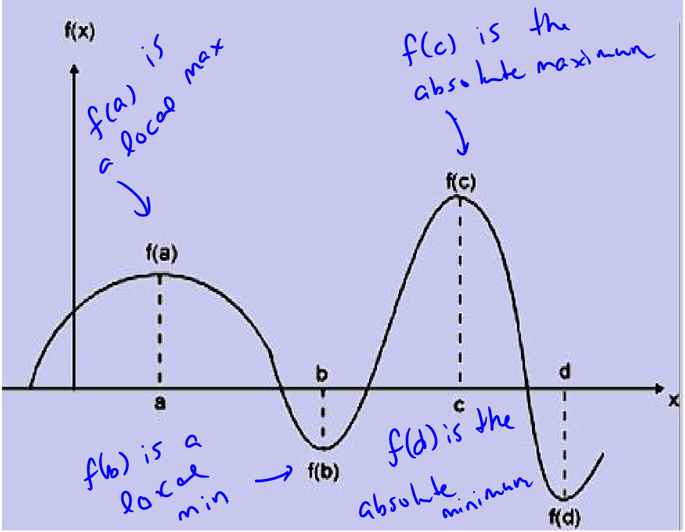


Figure: Graphically, local maxes and mins are *relative* high and low points.

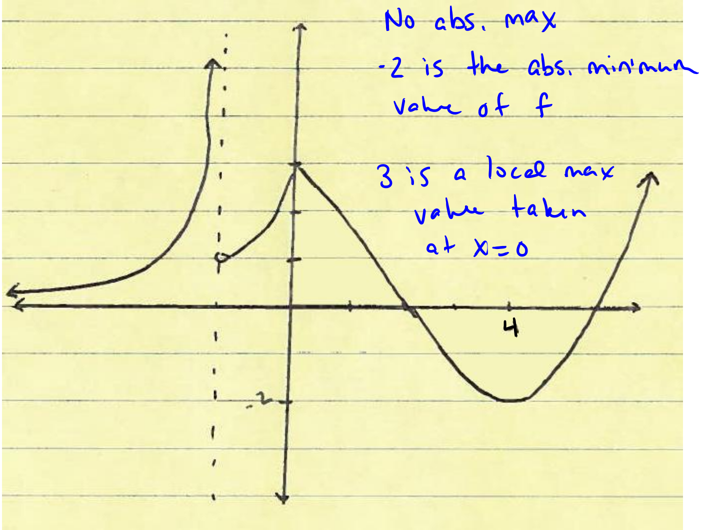


Figure: Identify local and absolute maxima and minima (if possible).

Terminology

Maxima—-plural of maximum

Minima—-plural of minimum

Extremum——is either a maximum or a minimum

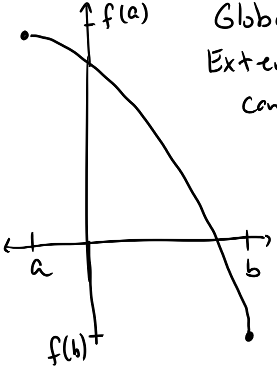
Extrema——plural of extremum

”**Global**” is another word for absolute.

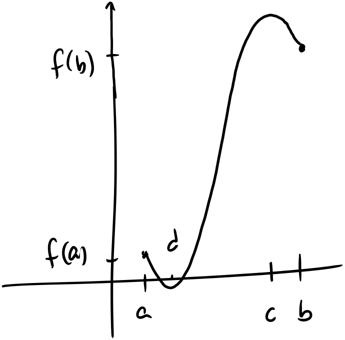
”**Relative**” is another word for local.

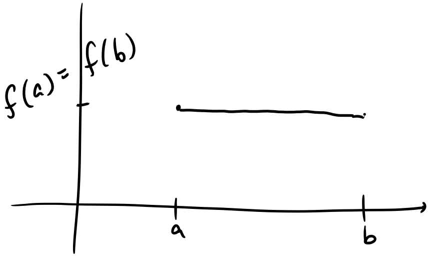
Extreme Value Theorem

Suppose f is continuous on a closed interval $[a, b]$. Then f attains an absolute maximum value $f(c)$ and f attains an absolute minimum value $f(d)$ for some numbers c and d in $[a, b]$.



Global Extreme can occur at end points

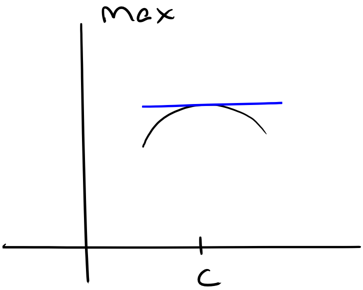



f(a)=f(b)
a
b

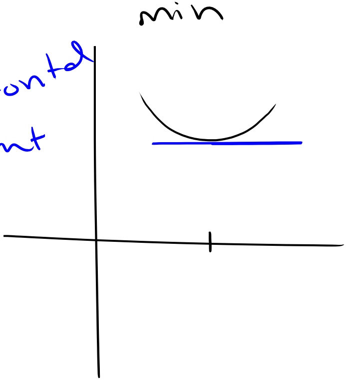
Fermat's Theorem

If f has a local extremum at c and if $f'(c)$ exists, then

$$f'(c) = 0.$$



horizontal tangent

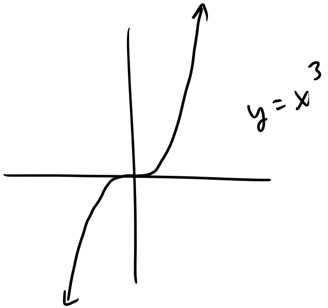


Question:

Suppose a function f satisfies $f'(0) = 0$. Can we conclude that $f(0)$ is a local maximum or local minimum?

No. Consider $f(x) = x^3$ so $f'(x) = 3x^2$

and $f'(0) = 0$

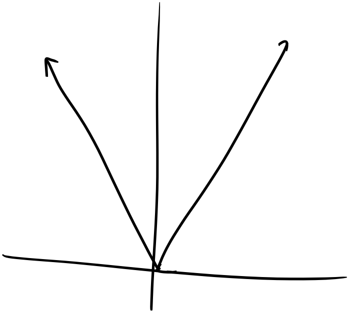


Question:

Could $f(c)$ be a local extremum but have $f'(c)$ not exist?

Yes. Consider $f(x) = |x|$

$f'(0)$ DNE



There is a global minimum value of zero @ $x = 0$.

Critical Number

Definition: A **critical number** of a function f is a number c in its domain such that either

$$f'(c) = 0 \quad \text{or} \quad f'(c) \text{ does not exist.}$$

Theorem: If f has a local extremum at c, then c is a critical number of f.

Critical numbers are also called critical points.