Sept 5 Math 2253H sec. 05H Fall 2014

Section 2.3: Some Differentiation Rules

$F(x)$	$F'(x)$	$F(x)$	$F'(x)$
c	0	$cf(x)$	$cf'(x)$
x	1	fg	$f'g+g'f$
x^n	nx^{n-1}	$\frac{f}{g}$	$\frac{f'g-g'f}{g^2}$

Example

Find $f''(x)$ given

$$f(x) = \sqrt[4]{x}$$

$$f(x) = x^{1/4}$$

Need f' first

$$f'(x) = \frac{1}{4} x^{\frac{1}{4}-1} = \frac{1}{4} x^{-3/4} = \frac{1}{4\sqrt[4]{x^3}}$$

$$f''(x) = \frac{1}{4}\left(\frac{-3}{4}\right) x^{-3/4-1}$$

$$= \frac{-3}{16} x^{-7/4} = \frac{-3}{16\sqrt[4]{x^7}}$$

Example

Find the equation of the line tangent to the graph of $y = \frac{1}{x^2+1}$ at the point $(-1, \frac{1}{2})$.

Need the slope: the slope $m = \frac{dy}{dx}$ @ $x = -1$

"$\left.\frac{dy}{dx}\right|_{x=-1}$" = "$\frac{dy}{dx}$ evaluated at $x=-1$"

$$\frac{dy}{dx} = \frac{0(x^2+1) - 1(2x)}{(x^2+1)^2} = \frac{-2x}{(x^2+1)^2}$$

$$m = \left.\frac{dy}{dx}\right|_{x=-1} = \frac{-2(-1)}{((-1)^2+1)^2} = \frac{1}{2}$$

The tangent line has equation

$$y - \frac{1}{2} = \frac{1}{2}(x - (-1))$$

$$\Rightarrow \quad \boxed{y = \frac{1}{2}x + 1}$$

Normal Line

The line *normal* to the graph of $y = f(x)$ at the point $(a, f(a))$ is the line perpendicular to the tangent line at that point.

Find the equation of the line normal to $y = \frac{1}{x^2+1}$ at the point $(-1, \frac{1}{2})$.

The slope of the tangent line was $\frac{1}{2}$,

the slope of the normal line is -2.

Normal line: $y - \frac{1}{2} = -2(x+1)$

$$\boxed{y = -2x - \frac{3}{2}}$$

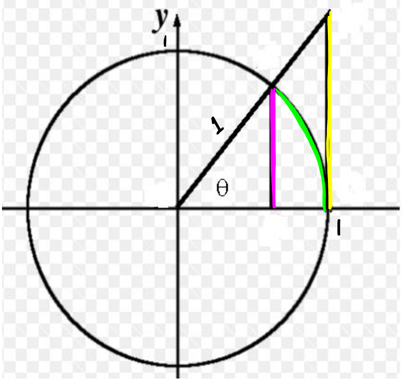
Section 2.4: Derivatives of Trigonometric Functions

Preliminary result:

$*\ s = r\theta\ *$

arc ↑ ↑ radius

$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1 \quad \text{and} \quad \lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta} = 0$$



- $\sin\theta$
- $\tan\theta$
- θ

$*\ 0 < \theta < \frac{\pi}{2}$

$$\underbrace{\sin\theta \le \theta} \le \tan\theta$$

$$\frac{\sin\theta}{\theta} \le 1$$

$$\theta \le \tan\theta \quad \Rightarrow \quad \theta \le \frac{\sin\theta}{\cos\theta} \quad \Rightarrow$$

$$\cos\theta \le \frac{\sin\theta}{\theta}$$

$$\cos\theta \le \frac{\sin\theta}{\theta} \le 1 \quad \text{for} \quad 0 < \theta < \frac{\pi}{2}$$

For $-\frac{\pi}{2} < \theta < 0$, $\quad 0 < -\theta < \frac{\pi}{2}$

$$\cos(-\theta) \le \frac{\sin(-\theta)}{-\theta} \le 1$$

by Symmetry $\quad \cos\theta \le \dfrac{-\sin\theta}{-\theta} \le 1$

$$\cos\theta \le \frac{\sin\theta}{\theta} \le 1 \quad \text{for } \theta \text{ in } \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$\theta \neq 0$$

$$\lim_{\theta \to 0} \cos\theta = \cos(0) = 1$$

$$\lim_{\theta \to 0} 1 = 1$$

$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$$

by the squeeze thm.

To get $\lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta}$

from the limit $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$

use the double (a.k.a half) angle ID

$$\cos\theta - 1 = 2\sin^2\left(\frac{\theta}{2}\right)$$