(1) A body is found at noon, and the coroner’s assistant determines the core body temperature to be 94.2°F. At 2pm, the coroner finally arrives and determines the core temperature of the body to be 84.6°F.

(a) Use a linear interpolation to approximate the temperature of the body at 1pm.

Let \( t = 0 \) correspond to noon. Then we have 2 data points \((t \sim \text{hrs}), (0, 94.2), (2, 84.6)\)

\[
P_1(t) = \frac{(t - t_0)y_1 + (t_1 - t)y_0}{t_1 - t_0} = \frac{1}{2} (84.6 + (2 - t) 94.2)
\]

\[
P_1(t) = -9.8t + 94.2
\]

At 1pm, the temperature is approximately

\[
P_1(.1) = -9.8 \times 1 + 94.2 = 84.4^\circ
\]

(b) Use the line found in part (a) to approximate the time of death. Assume the deceased’s body temperature was normal (98.6°F) at the time of death.

Let \( T \) be the time of death. Then

\[
98.6 = P_1(T) \Rightarrow T = \frac{98.6}{-9.8}
\]

\[
\frac{98.6}{-9.8} \times 60 \text{ minutes} = 55 \text{ min}
\]

This predicts a time of death at 11:05 am.
(c) The coroner notices that the assistant took a temperature reading at 1pm and recorded the true body temperature at that time to be 88.9°F. Use the data at noon, 1pm, and 2pm to find a quadratic interpolation approximating the body temperature.

Using three data points \((0, 94.2), (1, 88.9), (2, 84.6)\)

\[
L_0(t) = \frac{1}{2} (t-1)(t-2) \quad L_1(t) = -t(t-2) \quad L_2(t) = \frac{1}{2} t(t-1)
\]

\[
P_2(t) = 94.2 L_0(t) + 88.9 L_1(t) + 84.6 L_2(t)
\]

which simplifies to

\[
P_2(t) = 0.5 t^2 - 5.8 t + 94.2
\]

(d) What is the estimated time of death obtained using the quadratic polynomial?

For time \(T\) of death, \(P_2(T) = 98.6\)

\[
\Rightarrow T = -0.714599 \quad 60T = -42.88
\]

This model predicts a time of death of about 11:17 am.
(2) (a) Find the cubic polynomial that interpolates the following points.

\[ \{(-1, -8), (0, -1), (1, 2), (2, 7)\} \]

\[
L_0(x) = \frac{x(x-1)(x-2)}{-1(-2)(-3)} = \frac{1}{6} x(x-1)(x-2)
\]

\[
L_1(x) = \frac{(x+1)(x-1)(x-2)}{1(-1)(-2)} = \frac{1}{2} (x+1)(x-1)(x-2)
\]

\[
L_2(x) = \frac{(x+1)x(x-2)}{(2)(1)(-1)} = \frac{1}{2} x(x+1)(x-2)
\]

\[
L_3(x) = \frac{(x+1)x(x-1)}{3(2)(1)} = \frac{1}{6} x(x+1)(x-1)
\]

\[
P_3(x) = -8L_0(x) - L_1(x) + 2L_2(x) + 7L_3(x)
\]

which simplifies to

\[
P_3(x) = x^3 - 2x^2 + 4x - 1
\]

Note \[ P_3(3) = 27 - 18 + 12 - 1 = 20 \]

(b) Calling the polynomial you found \( P_3(x) \), show that \( P_3(3) = 20 \). Explain why no additional work is necessary to find the polynomial of degree at most four that interpolates the data.

\[ \{(-1, -8), (0, -1), (1, 2), (2, 7), (3, 20)\} \]

The polynomial of degree \( \leq 4 \) is unique. As \( P_3 \) passes through all those points, we'd find \( P_4(x) = P_3(x) \).