Home work 6: Due Thurs. March. 10, 2016 Math 2335 Spring 2016

Name: $\qquad$

(1) Consider the data in the table | $x_{n}$ | 1.1 | 1.2 | 1.3 |
| :---: | :---: | :---: | :---: |
| $f\left(x_{n}\right)$ | 0.1 | 0.22 | 0.40 |

(a) Compute the divided differences $f\left[x_{0}, x_{1}\right]$ and $f\left[x_{0}, x_{1}, x_{2}\right]$.
(b) Find the linear and quadratic interpolating polynomials using the Newton Divided Difference formulation.
(c) Suppose we know that the data comes from a function $f(x)$ that is twice differentiable and that $\left|f^{\prime \prime}(c)\right| \leq M$ for all $c$ in the interval $[1.1,1.3]$. Show that the error

$$
\left|f(x)-P_{1}(x)\right| \leq \frac{0.01 M}{8}, \quad \text { for } \quad 1.1 \leq x \leq 1.2
$$

(2) Let $f(x)=x^{2}$ for $0 \leq x \leq 1$. Compute the second order divided difference $f\left[x_{0}, x_{1}, x_{2}\right]$ for each set of nodes. Which theorem does this demonstrate?
(a) $\quad x_{0}=0, \quad x_{1}=\frac{1}{2}, \quad x_{2}=1$
(b) $\quad x_{0}=0, \quad x_{1}=\frac{1}{3}, \quad x_{2}=1$
(c) $\quad x_{0}=a, \quad x_{1}=b, \quad x_{2}=c \quad$ for any different numbers $a, b$ and $c$.

