January 10 MATH 1112 sec. 54 Spring 2020

Relations & Functions

We recall the following terminology associated with function notation:

- In $f(x)$, f is the function and x is its **argument**.
- x represents an element of the domain, $f(x)$ is an element of the range.
- Since $y = f(x)$, x is called the **independent variable** and y is called the **dependent variable**.
- $y = f(x)$ reads "y equals f of x"
- The collection of points $(x, f(x))$, for each x in the domain, is called **the graph of f**.

Graph of $f(x) = -x^2 + 2x + 4$

x	$f(x)$	$(x, f(x))$
$-\frac{3}{2}$	$-\frac{5}{4}$	$(-\frac{3}{2}, -\frac{5}{4})$
-1	1	$(-1, 1)$
0	4	$(0, 4)$
1	5	$(1, 5))$
$\frac{3}{2}$	$\frac{19}{4}$	$(\frac{3}{2}, \frac{19}{4})$
3	1	$(3, 1)$

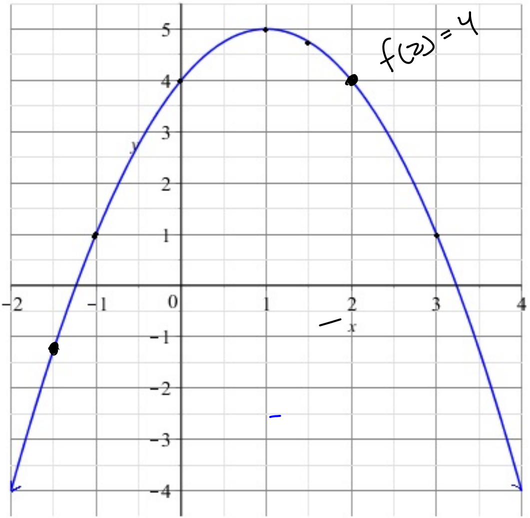
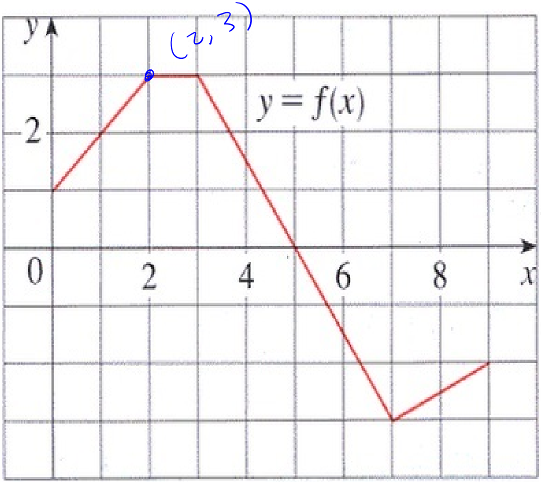


Figure: A table with several sample values and a graph of $y = f(x)$.

Question

From the graph of $y = f(x)$, evaluate $f(2)$



(a) 1

(b) 1 and 3.6

(c) 3

Vertical Line Test

The graph of a function can be intersected at most one time by any vertical line.

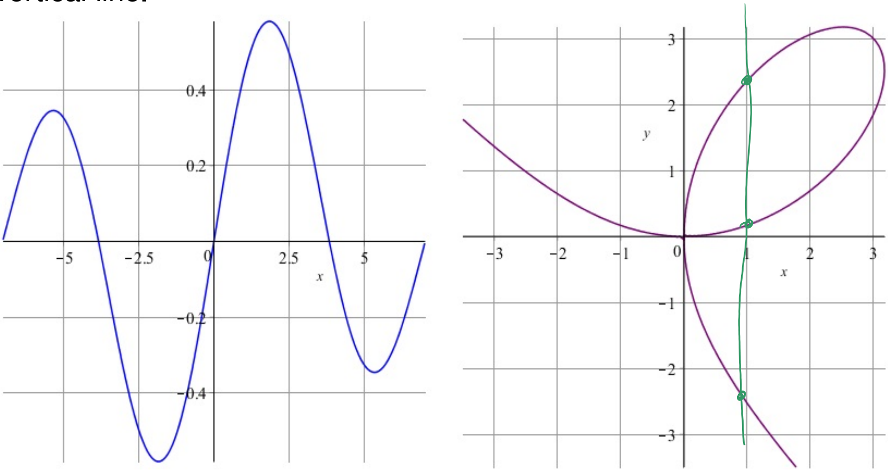


Figure: Plots of two relations. One is a function, the other is not.

Domain & Range

Unless stated otherwise, the domain of a function defined by an equation $y = f(x)$ is assumed to be the largest subset of the real numbers for which the value $f(x)$ is defined. In general, we eliminate any real numbers for which $f(x)$ is not defined as a real number. Recall

- division by zero is not defined
- negative numbers do not have any even roots (square root, fourth root, etc.)
- other *function properties* are (or will be) known such as negative numbers having no logarithms

Example

Determine the domain of

$$f(x) = \frac{\sqrt{x}}{x-1}$$

we'll determine what could not be in the domain.

We can't have $x-1=0$ (division by zero)

1 is not in the domain.

Due to $\sqrt{x}$ term, we need

x non negative.

In interval notation x nonnegative is

$x \geq 0$ $[0, \infty)$.

$$f(x) = \frac{\sqrt{x}}{x-1}$$

$x \neq 1$ in interval notation is

$(-\infty, 1) \cup (1, \infty)$

Satisfying both, the domain is

$[0, 1) \cup (1, \infty)$.

Question

The domain of $f(x) = \dfrac{1}{\sqrt{x+3}}$ is

(a) $(-3, \infty)$

(b) $(-2, 0) \cup (0, \infty)$

(c) $[-3, \infty)$

(d) $(-\infty, -3) \cup (-3, \infty)$

$\sqrt{x+3}$ requires

$x+3 \geq 0$

$x \geq -3$.

but

$\dfrac{1}{\sqrt{x+3}}$ requires

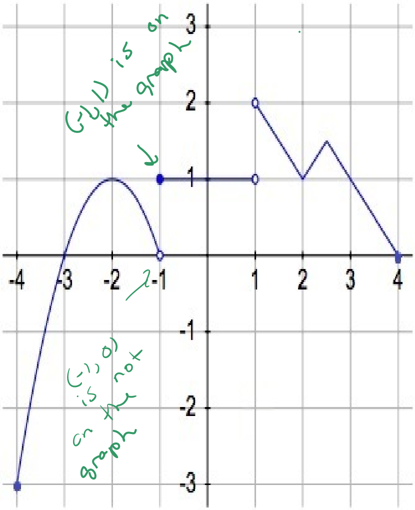
$x+3 > 0$

$x > -3$

Domain & Range

- The range may be difficult to infer from a formula. Sometimes it is possible by recalling known properties—e.g. $|x|$ is always nonnegative.
- The domain and range can often be determined from a graph.
- Recall that the range is the set of all possible $f(x)$—i.e.y—values.

Domain & Range from a Graph



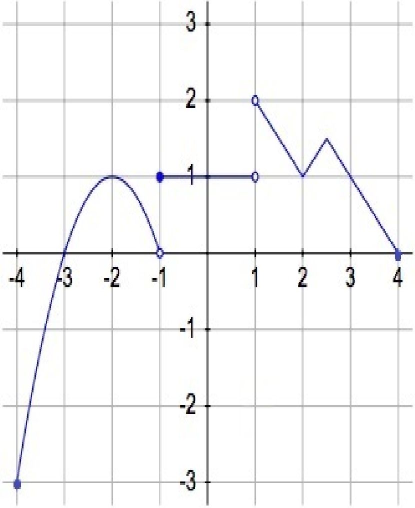
Observing x from -4 to 4, there is a y-value for each x except 1.

The domain is

$[-4, 1) \cup (1, 4]$.

Figure: Identify the domain from the plot $y = f(x)$

Domain & Range from a Graph



The lowest point is $(-4,-3)$. There is an open circle @ $(1,2)$.

There is curve for all y with

$-3 \le y < 2$.

In interval notation

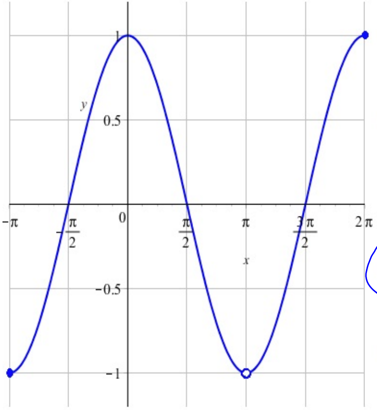
The range is

$[-3, 2)$.

Figure: Identify the range from the plot of $y = f(x)$

Question

Identify the domain and range from the graph of $y = f(x)$.



(a) Domain is $(-\pi, 2\pi)$, Range is $(-1, 1)$

(b) Domain is $[-\pi, 2\pi]$, Range is $[-1, 1]$

(c) Domain is $[-\pi, \pi) \cup (\pi, 2\pi]$, Range is $(-1, 1]$

(d) Domain is $[-\pi, \pi) \cup (\pi, 2\pi]$, Range is $[-1, 1]$

(e) can't be determined without more information