January 11 MATH 1112 sec. 54 Spring 2019

Two Equations in Two Variables (section 9.1)

We consider a system of two linear equations in two variables

$$\begin{array}{ccccc} ax & + & by & = & e \\ cx & + & dy & = & f \end{array}$$

We can try to solve this system using substitution, elimination, or graphing.

Two Equations in Two Variables

Theorem: Let a, b, c, d, e, and f be fixed constants. The system of equations

$$\begin{array}{ccccc} ax & + & by & = & e \\ cx & + & dy & = & f \end{array}$$

satisfies one of three cases:

- It has exactly one solution.
- It has infinitely many solutions.
- It has no solutions.

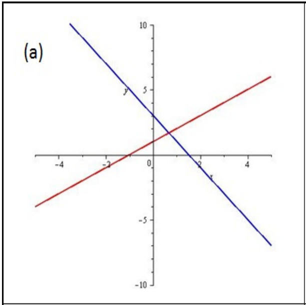
If the system has a solution (first two cases), it is called **consistent**. If it has no solutions, it is called **inconsistent**.

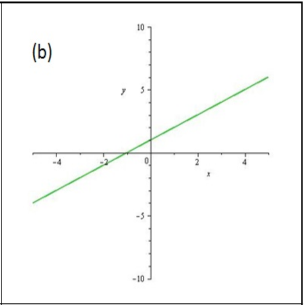
Consistent and Inconsistent Systems

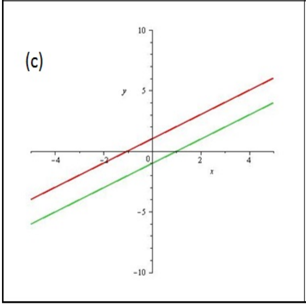
Consistent Independent: A system is called this when it has exactly one solution. Graphically, two lines intersect in one point.

Consistent Dependent: A system is called this when it has infinitely many solutions. Graphically, the equations define the same line. All points on that line represent solutions.

Inconsistent: A system is called this when it has no solutions. Graphically, the equations define distinct, parallel lines.







Graphical Illustration of Solution Cases

(a) $\begin{aligned} x - y &= -1 \\ 2x + y &= 3 \end{aligned}$ One solution case. Intersecting lines

(b) $\begin{aligned} x - y &= -1 \\ 2x - 2y &= -2 \end{aligned}$ Infinitely many solutions. One line

(c) $\begin{aligned} x - y &= -1 \\ 2x - 2y &= 2 \end{aligned}$ No solutions case. Parallel lines

Example

Determine if the system is consistent. If so, characterize the solution.

$$\begin{array}{rcrcl} 2x & + & y & = & 3 \\ x & + & \frac{1}{2}y & = & \frac{3}{2} \end{array}$$

using substitution

$2x + y = 3 \implies y = 3 - 2x$

Subbing in

$x + \frac{1}{2}(3 - 2x) = \frac{3}{2}$

$x + \frac{3}{2} - x = \frac{3}{2}$

$\frac{3}{2} = \frac{3}{2}$ this is always true!

This system is consistent, dependent.

The solutions are all the points on the line $y = -2x + 3$ (we got this from the 1st equation)

Question

Consider the system of equations

$$\begin{array}{rcrcl} x & - & y & = & 3 \\ 3x & + & y & = & 5 \end{array}$$

$x = 3 + y$

$3(3+y) + y = 5$

$9 + 3y + y = 5$

$4y = -4 \quad y = -1$

$x = 3 + (-1) = 2$

(a) This is consistent, independent with solutions $(4, 1)$.

(a) This is consistent, independent with solutions $(2, -1)$.

(a) This is consistent, dependent with infinitely many solutions.

(a) This is inconsistent.

Circles (Back to Section 1.1)

Definition: A **circle** is the set of all points in a plane equidistant from a fixed point called the center. The fixed distance is called the **radius.**

Equation of a circle: If the point (h, k) is the center of a circle of radius r in the Cartesian plane, then the set of points (x, y) on the circle satisfy

$$(x-h)^2+(y-k)^2=r^2.$$

The above is refered to as the **standard form** of the equation of the circle.

Question

$(x-h)^2 + (y-k)^2 = r^2$

The equation

$$(x-2)^2 + y^2 = 5$$

defines a circle with

$(x-2)^2 + (y-0)^2 = (\sqrt{5})^2$

(a) center $(-2, 0)$ and radius 5

(b) center $(0, 2)$ and radius $\sqrt{5}$

(c) center $(2, 0)$ and radius $\sqrt{5}$

(d) center $(2, 0)$ and radius 25

Example

Plot the circle whose points (x, y) satisfy the equation

$$x^2 + y^2 - 2x + 4y - 4 = 0$$

we need to know the center (h,k) and radius r.

we'll complete the square to write this as

$$(x-h)^2 + (y-k)^2 = r^2$$

$$\underbrace{x^2 - 2x}_{\text{add } 1} + \underbrace{y^2 + 4y}_{\text{add } 4} = 4$$

complete the squares

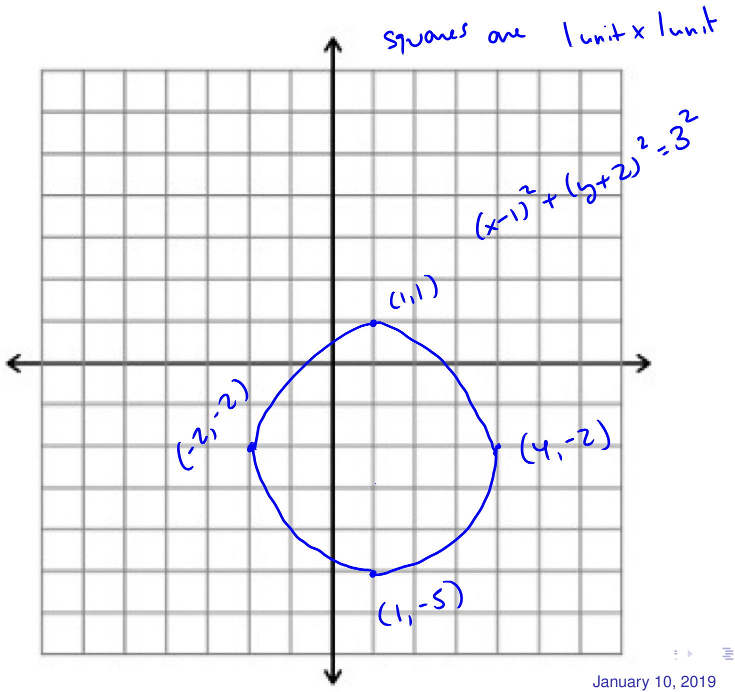
$$x^2 - 2x + 1 + y^2 + 4y + 4 = 4 + 1 + 4$$

$$(x-1)^2 + (y+2)^2 = 9$$

$$(x-1)^2 + (y+2)^2 = 3^2$$

$h = 1 \qquad k = -2 \qquad r = 3$

Center $(1,-2)$ radius 3


squares are 1 unit x 1 unit
$(x-1)^2+(y+2)^2=3^2$
(1,1)
(-2,-2)
(4,-2)
(1,-5)