January 14 MATH 1112 sec. 54 Spring 2019

Section 1.2: Relations & Functions

Given two sets of objects, we can define a **relation** on these sets.

Definition: A **relation** is a correspondence (or a mapping) between a first set, called the **domain**, and a second set, called the **range**, such that each element in the domain corresponds to (is related to/is mapped to) at least one element in the range.

Relations may be represented in several ways including

- lists of ordered pairs,
- equations in a pair of variables, and
- graphs.

Relation Examples

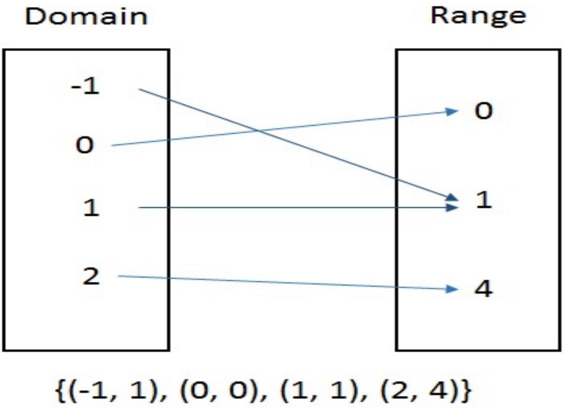

Figure: A relation on the sets $\{-1, 0, 1, 2\}$ and $\{0, 1, 4\}$ where the correspondence is that the range element is the square of the domain element. The relation is shown pictorially and as a set of ordered pairs. Note the order in the pairs is always (domain, range).

Relation Examples

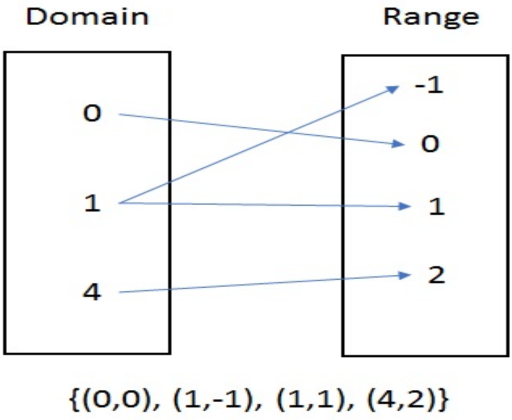

Figure: A relation on the sets $\{0, 1, 4\}$ and $\{-1, 0, 1, 2\}$ where the correspondence is that the range element is a number whose square is the domain element. The relation is shown pictorially and as a set of ordered pairs. Note the order in the pairs is always (domain, range).

Functions

These two examples highlight a characteristic of relations.

- Each domain element has at least one arrow emanating from it, and
- each range element has at least one arrow leading to it.

We have special names associated with restricting the number of such arrows. The most notable of these comes from insisting that each domain element have exactly one arrow coming from it.

Definition: A **function** on two sets is a relation in which each element of the domain corresponds to (is related to/is mapped to) exactly one element in the range.

Example

Function: The first relation $\{(-1,1),(0,0),(1,1),(2,4)\}$ is a function.
Note that no two ordered pairs have the same first element!

Not a Function: The second relation $\{(0,0),(1,-1),(1,1),(4,2)\}$ is NOT a function.
Note that two ordered pairs, $(1,-1)$ and $(1,1)$, have the same first element!

Observation: The definition of a function says that a domain element can only appear in one ordered pair. It does not restrict the appearance of a range element!

Question

Which (if any) of the following relations is a function?

(a) $\{(0,0),(1,1),(2,2)\}$

(b) $\{(0,1),(0,2),(0,3)\}$

(c) $\{(1,3),(-1,3),(7,7),(0,7)\}$

(d) (a) and (c)

(e) none of the above

Function Notation: An example

Consider the equation $y = 3x - 4$. We know that this equation defines a line in the plane. That is, it defines a set of points

$$(x, y) = (x, 3x - 4)$$

where x and y are elements of the set of real[1] numbers $\mathbb{R}$.

The equation $y = 3x - 4$ defines a function. Let's call this function f. We can express this in **function notation** as

$$f(x) = 3x - 4.$$

In English, this reads as

f of x equals three x minus 4.

[1] The symbol $\mathbb{R}$ denotes the set of all real number.

Function Notation: An example

$$\text{Let} \quad f(x) = 3x - 4, \quad \text{and suppose} \quad y = f(x)$$

- In $f(x)$, f is the function and x is its **argument**.
- x represents an element of the domain, $f(x)$ is an element of the range.
- Since $y = f(x)$, x is called the **independent variable** and y is called the **dependent variable**.
- $y = f(x)$ reads "y equals f of x"
- The collection of points $(x, f(x))$, for each x in the domain, is called **the graph of f**.

Example

Consider the function f defined by $f(x) = -x^2 + 2x + 4$. Evaluate each of the following.

(a) $f(-2) = -(-2)^2 + 2(-2) + 4 = -4 - 4 + 4 = -4$

(b) $f(3a) = -(3a)^2 + 2(3a) + 4 = -9a^2 + 6a + 4$

(c) $f(x+h) = -(x+h)^2 + 2(x+h) + 4$

$= -(x^2 + 2xh + h^2) + 2x + 2h + 4 = -x^2 - 2xh - h^2 + 2x + 2h + 4$

Question

Let $f(x) = -x^2 + 2x + 4$. Evaluate $f(3)$ and $f(-b)$.

$f(3) = -3^2 + 2(3) + 4 = -9 + 6 + 4 = 1$

(a) $f(3) = 1$ and $f(-b) = -3b + 4$

$f(-b) = -(-b)^2 + 2(-b) + 4$

(b) $f(3) = 7$ and $f(-b) = -b^2 + 2b + 4$

$= -b^2 - 2b + 4$

(c) $f(3) = 1$ and $f(-b) = b^2 - 2b + 4$

* note $-3^2 = -(3)^2 = -(3^2)$

(d) $f(3) = 1$ and $f(-b) = -b^2 - 2b + 4$

$-3^2 \neq (-3)^2$