January 15 MATH 1112 sec. 54 Spring 2020

Analyzing Graphs and Piecewise Defined Functions

Sometimes we want to consider functions that are defined by different rules over different parts of the domain. Such a function can be called a **piecewise defined function**.

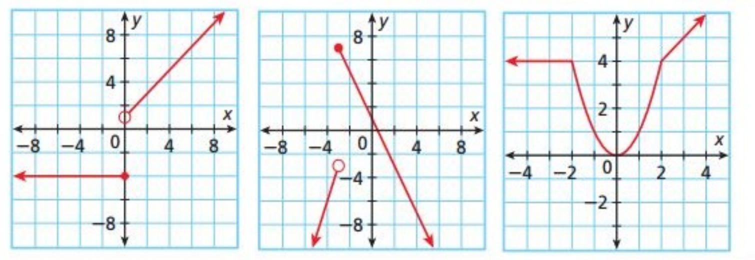


Figure: Examples of plots of piecewise defined functions. Note that the pieces may or may not be connected, but the graph of a function must still satisfy the vertical line test.

Notation for Piecewise Defined Functions

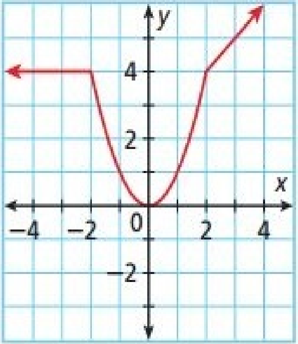
Suppose a function f has domain $(-\infty, \infty)$ and is defined so that

- For $-\infty < x < -2$, the rule for f is $f(x) = 4$;
- for $-2 \le x < 2$, the rule for f is $f(x) = x^2$; and
- for $x \ge 2$, the rule for f is $f(x) = x + 2$.

A standard way of expressing this function is

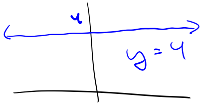
$$f(x) = \begin{cases} 4, & x < -2 \\ x^2, & -2 \le x < 2 \\ x + 2, & x \ge 2 \end{cases}$$

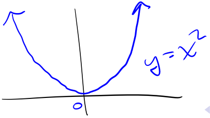
Piecewise Defined Function with Graph

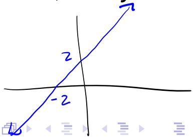


$$f(x) = \begin{cases} 4, & x < -2 \\ x^2, & -2 \leq x < 2 \\ x + 2, & x \geq 2 \end{cases}$$

Figure: Note that pieces of the graph appear as $y = 4$, $y = x^2$, and $y = x + 2$ over the specified intervals.







Evaluating Piecewise Defined Functions

Determine the appropriate *rule* according to the definition, then obtain a function value.

$$\text{Let } f(x) = \begin{cases} 4, & x < -2 \\ x^2, & -2 \leq x < 2 \\ x + 2, & x \geq 2 \end{cases}$$

Evaluate

(a) $f(3) = 3+2 = 5$

$3 \geq 2$

(b) $f(-3) = 4$

$-3 < -2$

(c) $f(-1) = (-1)^2 = 1$

$-2 \leq -1 < 2$

(d) $f\left(\frac{15}{7}\right) = \frac{15}{7} + 2 = \frac{29}{7}$

$\frac{15}{7} > \frac{14}{7} = 2$

Evaluting Piecewise Defined Functions

Let $f(x) = \begin{cases} 4, & x < -2 \\ x^2, & -2 \leq x < 2 \\ x + 2, & x \geq 2 \end{cases}$

Suppose h is a small positive number (say $0 < h < 0.1$). Evaluate

(a) $f(-2+h)$ $= (-2+h)^2 = 4 - 4h + h^2$

Since $0 < h < 0.1$ $\quad -2 \leq -2+h < 2$

(b) $f(-2-h)$ $= 4$

$h > 0$ so $-2-h < -2$

(c) $f(2+h)$ $= (2+h) + 2 = 4+h$

$h > 0$ so $2+h \geq 2$

Plotting Piecewise Defined Functions

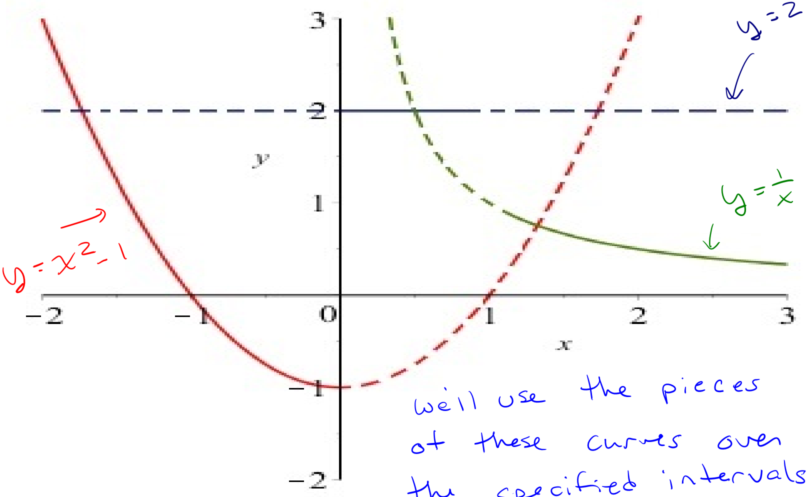
If we know how to plot the different pieces of a piecewise defined function, then we can sketch its plot. As an example, let's plot

$$f(x) = \begin{cases} x^2 - 1, & x < 0 \\ 2, & 0 \le x < 1 \\ \frac{1}{x}, & x \ge 1 \end{cases}$$

We can start by looking at the graphs of each of

$$y = x^2 - 1, \quad y = 2, \quad \text{and} \quad y = \frac{1}{x}$$

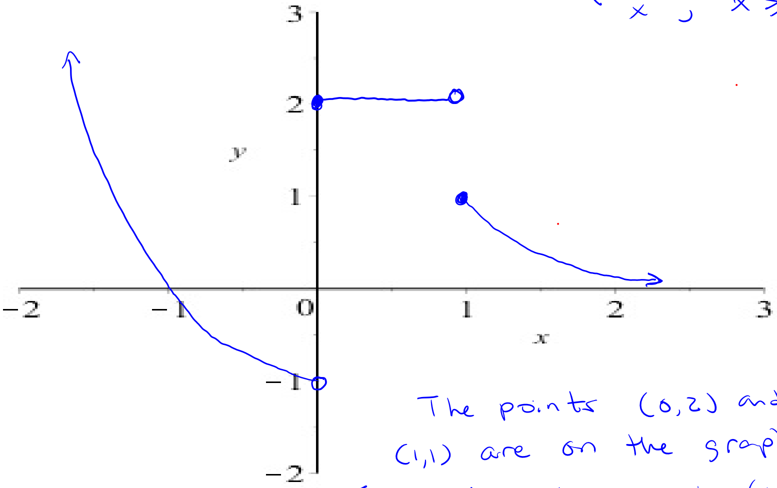
Plotting Example



we'll use the pieces of these curves over the specified intervals.

Plotting Example

$$y = f(x) = \begin{cases} x^2 - 1, & x < 0 \\ 2, & 0 \le x < 1 \\ \frac{1}{x}, & x \ge 1 \end{cases}$$



The points $(0,2)$ and $(1,1)$ are on the graph.

The points $(0,-1)$ and $(1,2)$ are not.