January 14 MATH 1112 sec. 54 Spring 2019

Section 1.2: Relations & Functions

Function Notation: An example

$$\text{Let} \quad f(x) = 3x - 4, \quad \text{and suppose} \quad y = f(x)$$

- In $f(x)$, f is the function and x is its **argument**.

- x represents an element of the domain, $f(x)$ is an element of the range.

- Since $y = f(x)$, x is called the **independent variable** and y is called the **dependent variable**.

- $y = f(x)$ reads "y equals f of x"

- The collection of points $(x, f(x))$, for each x in the domain, is called **the graph of f**.

Graph of $f(x) = -x^2 + 2x + 4$

x	$f(x)$	$(x, f(x))$
$-\frac{3}{2}$	$-\frac{5}{4}$	$(-\frac{3}{2}, -\frac{5}{4})$
-1	1	$(-1, 1)$
0	4	$(0, 4)$
1	5	$(1, 5))$
$\frac{3}{2}$	$\frac{19}{4}$	$(\frac{3}{2}, \frac{19}{4})$
3	1	$(3, 1)$

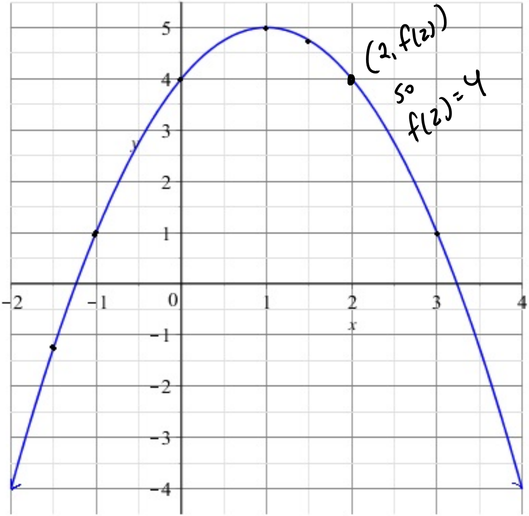
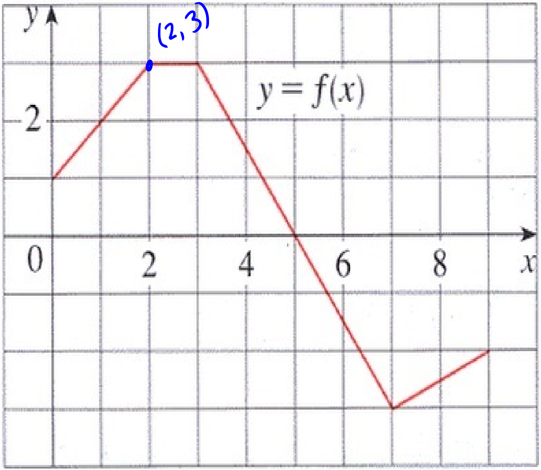


Figure: A table with several sample values and a graph of $y = f(x)$.

Question

From the graph of $y = f(x)$, evaluate $f(2)$



(a) 1

(b) 1 and 3.6

(c) 3

Vertical Line Test

The graph of a function can be intersected at most one time by any vertical line.

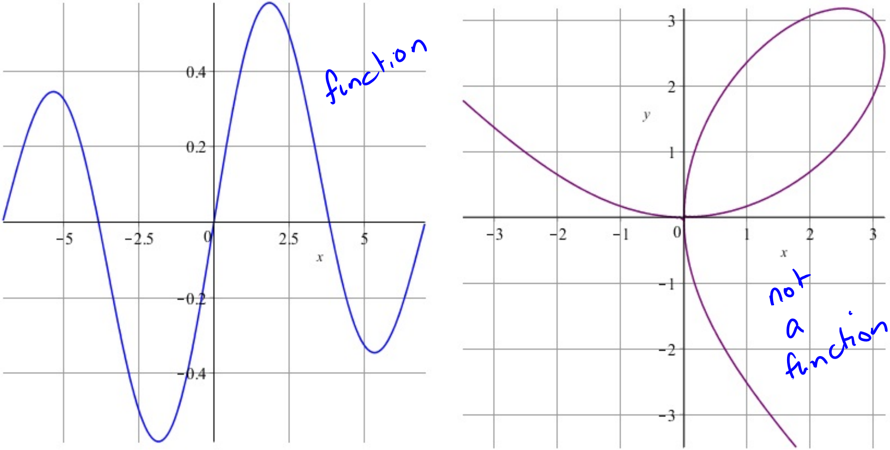


Figure: Plots of two relations. One is a function, the other is not.

Domain & Range

Unless stated otherwise, the domain of a function defined by an equation $y = f(x)$ is assumed to be the largest subset of the real numbers for which the value $f(x)$ is defined. In general, we eliminate any real numbers for which $f(x)$ is not defined as a real number. Recall

- division by zero is not defined
- negative numbers do not have any even roots (square root, fourth root, etc.)
- other *function properties* are (or will be) known such as negative numbers having no logarithms

Example

Determine the domain of the function.

$$f(x) = \frac{\sqrt{x}}{x-1}$$

We identify what is not in the domain.

We require $\sqrt{x}$ defined. So $x \geq 0$.

We require $x - 1 \neq 0$ to avoid division by zero. So $x \neq 1$.

The domain is all $x \geq 0$ such that $x \neq 1$.

In interval notation, the domain is

$$[0, 1) \cup (1, \infty).$$

Question

The domain of $f(x) = \dfrac{x^2}{\sqrt[4]{x+3}}$ is

(a) $(-3, \infty)$

(b) $(-2, 0) \cup (0, \infty)$

(c) $[-3, \infty)$

(d) $(-\infty, -3) \cup (-3, \infty)$

The possible problems are negative under the 4th root and zero in the denominator.

we require

$x + 3 > 0$

$x > -3$

Domain & Range

- The range may be difficult to infer from a formula. Sometimes it is possible by recalling known properties—e.g. $|x|$ is always nonnegative.
- The domain and range can often be determined from a graph.
- Recall that the range is the set of all possible $f(x)$—i.e.y—values.

Domain & Range from a Graph

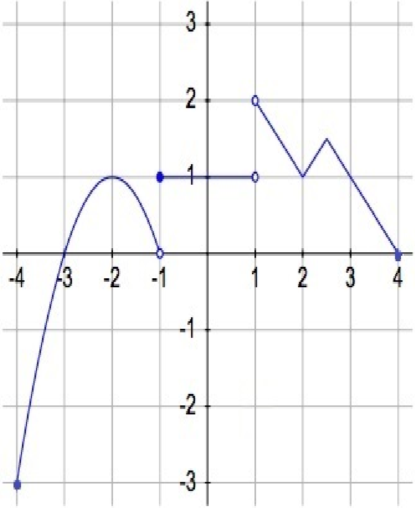


Figure: Identify the domain and range from the plot of $y = f(x)$

The domain is

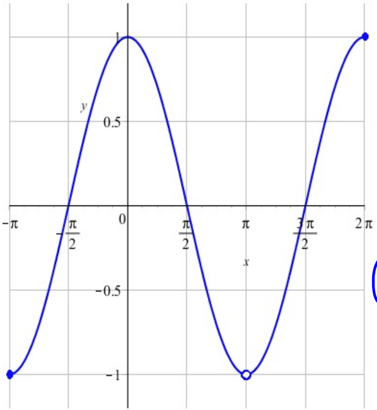
$[-4, 1) \cup (1, 4]$

The range is

$[-3, 2)$.

Question

Identify the domain and range from the graph of $y = f(x)$.



(a) Domain is $(-\pi, 2\pi)$, Range is $(-1, 1)$

(b) Domain is $[-\pi, 2\pi]$, Range is $[-1, 1]$

(c) Domain is $[-\pi, \pi) \cup (\pi, 2\pi]$, Range is $(-1, 1]$

(d) Domain is $[-\pi, \pi) \cup (\pi, 2\pi]$, Range is $[-1, 1]$

(e) can't be determined without more information

Section 2.2: The Algebra of Functions

Calculus Alert: Being fluent in the operations and the notation will help you be successful in Calculus.

We can create new functions from old by combining them with the operations of addition, subtraction, multiplication, division, and composition.

Addition, Subtraction, Multiplication, and Division of Functions

Let f and g be functions, and suppose that x is in the domain of each. Then define $f+g$, $f-g$, fg and f/g, and use the following notation

- $(f+g)(x) = f(x) + g(x)$
- $(f-g)(x) = f(x) - g(x)$
- $(fg)(x) = f(x)g(x)$
- $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ provided $g(x) \neq 0$

Domain

For functions f and g, the domain of $f+g$, $f-g$, and fg is the set of all x such that x is in the domain of f and x is in the domain of g.

The domain of f/g is the set of all all x such that x is in the domain of f, x is in the domain of g, and $g(x) \neq 0$.

Example

Let $f(x) = \sqrt{x+1}$ and $g(x) = 3x^2$. Evaluate

(a) $(f+g)(3)$ $= f(3) + g(3) = \sqrt{3+1} + 3(3^2) = \sqrt{4} + 27 = 29$

(b) $(f-g)\left(-\frac{3}{4}\right)$ $= f\left(\frac{-3}{4}\right) - g\left(\frac{-3}{4}\right) = \sqrt{\frac{-3}{4}+1} + 3\left(\frac{-3}{4}\right)^2 = \sqrt{\frac{1}{4}} + 3\left(\frac{9}{16}\right)$

$= \frac{1}{2} + \frac{27}{16} = \frac{8+27}{16} = \frac{35}{16}$

(c) $(f+g)(x)$

$= f(x) + g(x) = \sqrt{x+1} + 3x^2$