January 17 Math 1190 sec. 63 Spring 2017

Second Day of Class

Today's Agenda

- Questions?
- Clicker activities
- Up coming Exam 1 (part 1) information
- More prerequisite review
- Section 1.1: Limits
- Any announcements from our SI leader Norman

Registering a Clicker

At the beginning of class, I will use the ”Roll Call” feature. You will see your name and student ID with a three letter code.

- Grab a clicker from my stash at the beginning of class.
- Look for your name with three letter code on the roll call display. (All names won’t fit on one screen, so it will alternate between groups.)
- Turn the clicker on, and methodically enter your three letter code.
- When your clicker is registered, your name box will turn gray with an ID code in the bottom right corner.
- If you press the wrong code, no worries, just press ”DD” (or ”DDD”).

Clicker Questions

Sample Question 1

The line $y = 3x + 1$ is **perpendicular** to which of the following lines.

(a) $y = -3x - 1$

(b) $y = \frac{1}{3}x - 1$

(c) $y = -\frac{1}{3}x + 4$

(d) $y = x - 3$

(e) None of the above

Slopes of perpendicular lines are negative reciprocals.

Sample Question 2

Recall that for True/False questions, we'll always use "A" for true and "B" for false.

True/False If f is a one-to-one function satisfying $f(2) = -3$, then

$$f^{-1}(-3) = 2.$$

This is the function / inverse function relationship.

Sample Question 3

The quadratic equation $x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

(a) has solutions $x = 4$ and $x = -2$

$x+4=0 \Rightarrow x=-4$

or

(b) has solutions $x = 2$ and $x = -4$

$x-2=0 \Rightarrow x=2$

(c) has solutions $x = -2$ and $x = 8$

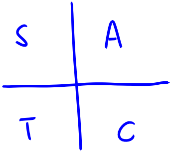
(d) has no real solutions.

Sample Question 4

Suppose θ is an angle in standard position, and that

$$\sin\theta < 0 \quad \text{and} \quad \tan\theta > 0.$$

The terminal side of θ must be in quadrant



$\sin\theta < 0$ in quads III and IV

$\tan\theta > 0$ in quads I and III

(a) I (one)

(b) II (two)

(c) III (three)

(d) IV (four)

(e) can't be determined without more information

Exam 1 part 1

When: Thursday January 19 (two days from now) from 10:05am–10:30am (25 minutes)

What: This exam will make up 35% of Exam 1 for the semester. It will cover prerequisite topics: Algebra, trigonometry, and function basics. Worksheets 1, 2, and 3 in D2L and the course page cover this material.

Why: The two main causes of poor performance in Calculus are (1) prerequisite weakness, and (2) insufficient effort. The beginning of the semester is the time to hone those prereq skills and position yourself for success in this class.

Simplify Each Expression

(a) $\ln(e^8) = 8$

$= 8 \ln e = 8 \cdot 1 = 8$

(b) $e^{2\ln 7} = e^{\ln 7^2} = e^{\ln 49} = 49$

$= \left(e^{\ln 7}\right)^2 = 7^2 = 49$

Properties:

$\ln e^x = x$ for all x

$e^{\ln x} = x$ for all $x > 0$

$\ln a^r = r \ln a$

$\ln e = 1$

$\ln 1 = 0$

$a^{b \cdot c} = (a^c)^b$

Factor completely

$$4(x+3)^3(x-1)^{-2} - 2(x+3)^4(x-1)^{-3}$$

$$= (x+3)^3\left(4(x-1)^{-2} - 2(x+3)(x-1)^{-3}\right)$$

$$= (x+3)^3(x-1)^{-3}\left(4(x-1)^{1} - 2(x+3)\right)$$

$$= (x+3)^3(x-1)^{-3}(4x-4-2x-6)$$

$$= (x+3)^3(x-1)^{-3}(2x-10)$$

$$= 2(x+3)^3(x-1)^{-3}(x-5)$$

This can be left as is or written as

$$\frac{2(x+3)^3(x-5)}{(x-1)^3}$$

Let $f(x) = 2 - 4x^2$. Evaluate and simplify

$$\frac{f(1+h) - f(1)}{h}$$

$$f(1) = 2 - 4(1^2) = -2$$

$$f(1+h) = 2 - 4(1+h)^2 = 2 - 4\left(1 + 2h + h^2\right)$$

$$= 2 - 4 - 8h - 4h^2 = -2 - 8h - 4h^2$$

so $$\frac{f(1+h) - f(1)}{h} = \frac{-2 - 8h - 4h^2 - (-2)}{h}$$

$$= \frac{-\cancel{2} - 8h - 4h^2 + \cancel{2}}{h}$$

$$= \frac{-8h - 4h^2}{h}$$

$$= \frac{-4\cancel{h}(2+h)}{\cancel{h}} = -4(2+h)$$

Consider the piecewise defined function

$$f(x) = \begin{cases} \frac{1}{x}, & x \leq -1 \\ x^2 - 1, & x > -1 \end{cases}$$

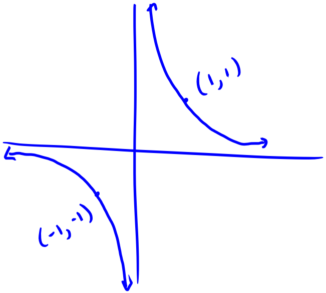
Evaluate

- $f(-1) =$ $\frac{1}{-1} = -1$
- $f(0) =$ $0^2 - 1 = -1$
- $f(1) =$ $1^2 - 1 = 0$

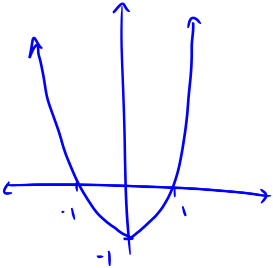
Produce a rough plot of f.

$$f(x) = \begin{cases} \frac{1}{x}, & x \leq -1 \\ x^2 - 1, & x > -1 \end{cases}$$

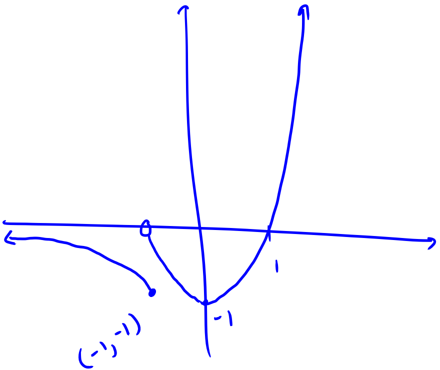
$y = \frac{1}{x}$



$y = x^2 - 1$



$$f(x) = \begin{cases} \frac{1}{x}, & x \leq -1 \\ x^2 - 1, & x > -1 \end{cases}$$



Section 1.1: Limits of Functions Using Numerical and Graphical Techniques

In *Calculus*, we consider the way in which quantities **change.** In particular, if we have a function representing some process (motion of a particle, growth of a population, spread of a disease), we can analyze it to determine the nature of how it changes. We can also use knowledge of change to reconstruct a function describing a process.

Central to analyzing change and reconstructing functions is notion of a **limit.**

Slope

We know that a non-vertical line has the form

$$y = mx + b.$$

The slope m tells us how the dependent variable y will change if the independent variable x changes by a set amount Δx.

Slope

Consider $y = 3x - 1$ which has slope $m = 3$.

Note that two points on this line are $(1, 2)$ and $(3, 8)$. The change in x between these two points is

$$\Delta x = 3 - 1 = 2.$$

Compute the change in y, Δy.

$$\Delta y = 8 - 2 = 6 = 3 \cdot 2 = 3\Delta x = m\Delta x$$

note $\quad m\Delta x = \Delta y \implies m = \dfrac{\Delta y}{\Delta x}$

Question

The slope of the line containing the points $(2, 2)$ and $(3, 7)$ is

$\Delta y = 7 - 2 = 5$

$\Delta x = 3 - 2 = 1$

(a) $m = \frac{1}{5}$

(b) $m = 5$

$m = \frac{\Delta y}{\Delta x} = \frac{5}{1} = 5$

(c) $m = \frac{7}{2}$

(d) can't be determined without more information

Slope of a general curve

So slope of a line tells us how y changes when x changes. What if the curve isn't a line??

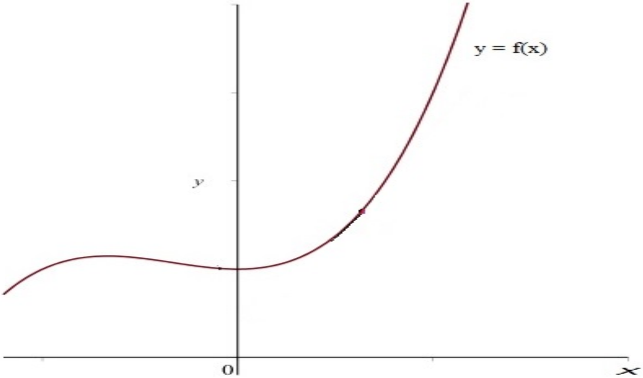


Figure: Can we consider *slope* for a curve like this?

The Tangent Line Problem

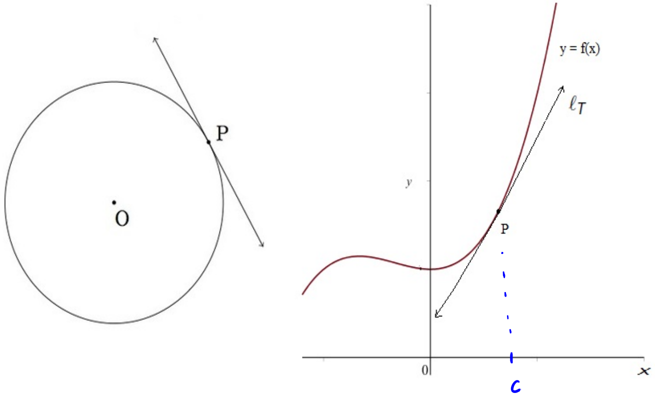


Figure: We begin by considering the tangent line problem. For a circle, a tangent line at point P is defined as the line having exactly one point in common with the circle. For the graph of a function $y = f(x)$, we define the tangent line at the point P has the line that shares the point P and has the same *slope* as the graph of f at P.

Slope of the Tangent Line

Question: What is meant by the *slope* of the function at the point P?

For now, let's assume that the graph if reasonably *nice* like the one in the figure. Let P be at $x = c$ and $y = f(c)$

$$\text{i.e.} \quad P = (c, f(c)).$$

To find a slope, we require two points. So let's take another point Q on the graph of f. In term of coordinates

$$Q = (x, f(x)).$$

The line through the two points P and Q on the graph is called a **Secant Line**. We will denote the slopes of the tangent line and the secant line as

$$m_{tan} \quad \text{and} \quad m_{sec}.$$

Slope of the Tangent Line

We have two points on our curve $y = f(x)$. The points at c and x (with $x \neq c$) are

$$P = (c, f(c)), \quad \text{and} \quad Q = (x, f(x)).$$

Compute the slope m_{sec} of the secant line through these points.

$$\Delta y = f(x) - f(c) \qquad \Delta x = x - c$$

$$m_{sec} = \frac{\Delta y}{\Delta x} = \frac{f(x) - f(c)}{x - c}$$

Slope of the Tangent Line

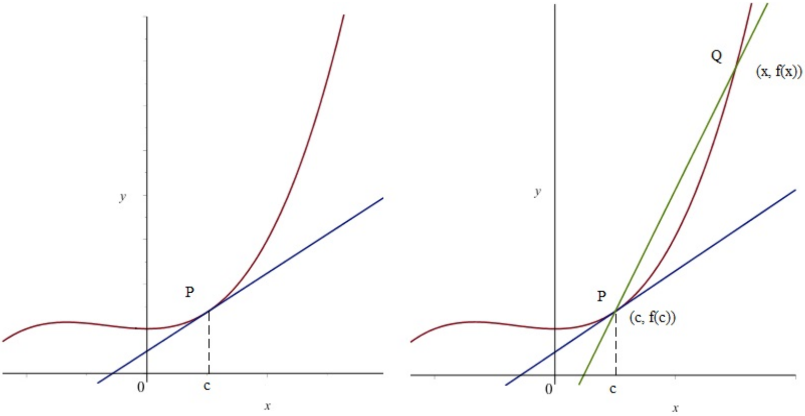
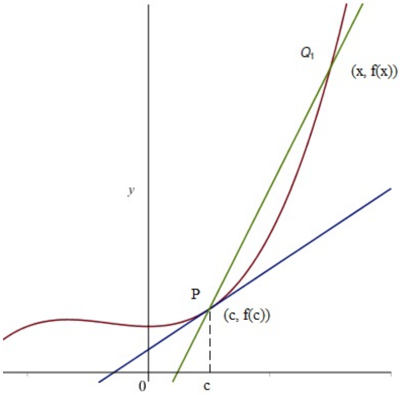


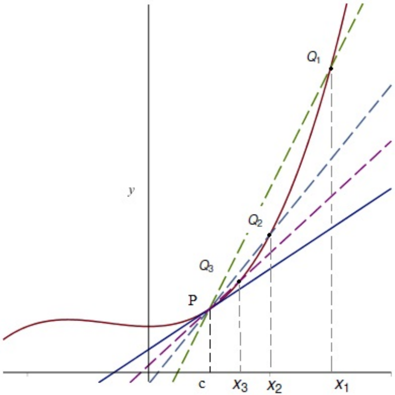
Figure: The slope of the line through P and Q (rise over run) is

$$m_{sec} = \frac{f(x) - f(c)}{x - c}$$

Slope of the Tangent Line

We consider a sequence of points $Q_1 = (x_1, f(x_1))$, $Q_2 = (x_2, f(x_2))$, and so forth in such a way that the x-values are getting closer to c. Note that the resulting secant lines tend to have slopes closer to that of the tangent line.





Slope of the Tangent Line

We call this process a *limit*. We will define the slope of the tangent line as

$$m_{tan} = \left[\text{Limit of} \quad \frac{f(x) - f(c)}{x - c} \quad \text{as } x \text{ gets closer to } c\right].$$

Our notation for this process will be

$$m_{tan} = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}.$$

The notation $\lim_{x \to c}$ reads as "the limit as x approaches c."

Notation: The notation $\lim_{x \to c}$ is always followed by an algebraic expression. It is never immediately followed by an equal sign.

A Working Definition of a Limit

Definition: Let f be defined on an open interval containing the number c except possibly at c. Then

$$\lim_{x \to c} f(x) = L$$

provided the value of $f(x)$ can be made arbitrarily close to the number L by taking x sufficiently close to c but not equal to c.

$x = c$ is not permitted in this definition.

L is associated w/ "y-values"

c is associated with "x-values"

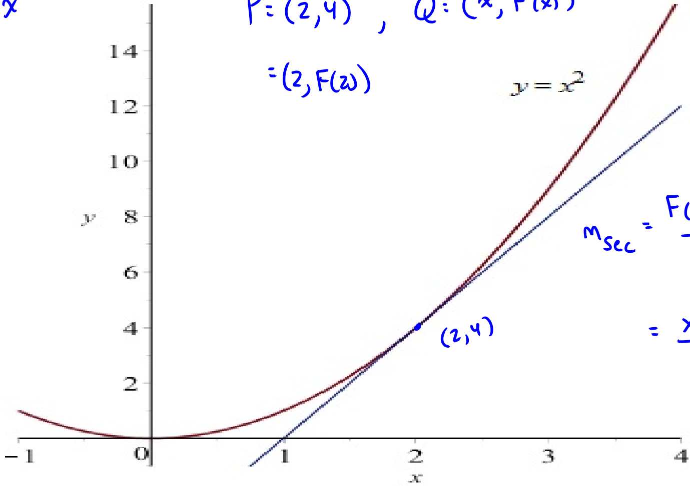
Example

Use a calculator to determine the slope of the line tangent to the graph of $y = x^2$ at the point $(2, 4)$.

let $F(x) = x^2$

$P : (2,4)$, $Q : (x, F(x))$

$= (2, F(2))$



$$m_{sec} = \frac{F(x) - F(2)}{x - 2}$$

$$= \frac{x^2 - 4}{x - 2}$$

$$m_{tan} = \lim_{x\to 2} \frac{F(x)-F(2)}{x-2} = \lim_{x\to 2} \frac{x^2-4}{x-2}$$

x	$\frac{F(x)-F(2)}{x-2}$
1.9	$\frac{F(1.9)-F(2)}{1.9-2} = \frac{(1.9)^2-4}{1.9-2} = 3.9$
1.99	3.99
1.999	3.999
2	undefined
2.001	4.001
2.01	4.01
2.1	$\frac{(2.1)^2-4}{2.1-2} = 4.1$

It appears like

$$\lim_{x\to 2} \frac{x^2-4}{x-2} = 4$$

In fact $m_{tan} = 4$

Example

Use a calculator and table of values to investigate

$$\lim_{x \to 0} \frac{e^x - 1}{x}$$

x	$f(x) = \frac{e^x - 1}{x}$
-0.1	$(e^{-0.1}-1)/(-0.1) \approx 0.9516$
-0.01	0.9950
-0.001	0.9995
0	undefined
0.001	1.0005
0.01	1.0050
0.1	$(e^{0.1}-1)/0.1 \approx 1.0517$

looks like

$$\lim_{x \to 0} \frac{e^x - 1}{x} = 1$$

Question

True or False: In order to evaluate $\lim_{x \to c} f(x)$, the value of $f(c)$ must be defined (i.e. c must be in the domain of f)?

f(c) need not be defined.