January 22 MATH 1112 sec. 54 Spring 2020

The Algebra of Functions

We can create new functions from old by combining them with the operations of addition, subtraction, multiplication, division, and composition.

Most of the operations have intuitive definitions. We want to focus on understanding how to read and write the notation.

Addition, Subtraction, Multiplication, and Division of Functions

Let f and g be functions, and suppose that x is in the domain of each. Then define $f+g$, $f-g$, fg and f/g, and use the following notation

- $(f+g)(x) = f(x) + g(x)$ *("f plus g of x is f of x plus g of x")*
- $(f-g)(x) = f(x) - g(x)$
- $(fg)(x) = f(x)g(x)$
- $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ provided $g(x) \neq 0$

Domain

For functions f and g, the domain of $f+g$, $f-g$, and fg is the set of all x such that x is in the domain of f and x is in the domain of g.

x is in the domain of $f+g$ if x is in the domain of f, AND x is in the domain of g.

The domain of f/g is the set of all all x such that x is in the domain of f, x is in the domain of g, and $g(x) \neq 0$.

Example

Let $f(x) = \sqrt{x+1}$ and $g(x) = 3x^2$. . Evaluate

(a) $(f+g)(3)$ $= f(3) + g(3) = \sqrt{3+1} + 3(3^2) = 2 + 27 = 29$

(b) $(f-g)\left(-\frac{3}{4}\right) = f\left(\frac{-3}{4}\right) - g\left(\frac{-3}{4}\right) = \sqrt{\frac{-3}{4}+1} - 3\left(\frac{-3}{4}\right)^2$

$= \sqrt{\frac{1}{4}} - 3\left(\frac{9}{16}\right) = \frac{1}{2} - \frac{27}{16} = \frac{8-27}{16} = \frac{-19}{16}$

(c) $(fg)(x)$ $= f(x)g(x) = \sqrt{x+1}\,(3x^2) = 3x^2\sqrt{x+1}$

Example: $f(x) = \sqrt{x+1}$ and $g(x) = 3x^2$

Note that

$$\text{Domain of } f = [-1, \infty), \quad \text{and} \quad \text{Domain of } g = (-\infty, \infty).$$

Identify the domain of $\dfrac{f}{g}$.

we need all x in both domains and such that

$g(x) \neq 0$.

Note $g(x) = 0 \Rightarrow 3x^2 = 0 \Rightarrow x = 0$

So zero is <u>not</u> in the domain of $\frac{f}{g}$.

Since all x in $[-1, \infty)$ is in $(-\infty, \infty)$,

Our domain will be all of $[-1, \infty)$

except $x = 0$.

In interval notation, the domain of $\frac{f}{g}$ is $[-1, 0) \cup (0, \infty)$.

Question

Let $f(x) = 3x^2$ and $g(x) = \dfrac{2}{x-5}$. Evaluate $(fg)(2)$.

(a) $(fg)(2) = 8$

(b) $(fg)(2) = -8$

(c) $(fg)(2) = -12$

(d) $(fg)(2) = 12$

$$(fg)(2) = f(2)\,g(2)$$

$$= 3(2^2)\left(\frac{2}{2-5}\right)$$

$$= 3(4)\left(\frac{-2}{3}\right) = -8$$

Question

Let $f(x) = 3x^2$ and $g(x) = \dfrac{2}{x-5}$. Evaluate $\left(\dfrac{f}{g}\right)(1)$.

(a) $\left(\dfrac{f}{g}\right)(1) = -\dfrac{2}{3}$

$$\left(\frac{f}{g}\right)(1) = \frac{f(1)}{g(1)} = \frac{3(1^2)}{\frac{2}{1-5}}$$

(b) $\left(\dfrac{f}{g}\right)(1) = 6$

$$= \frac{3}{\frac{2}{-4}} = \frac{3}{-1/2} = 3\left(\frac{-2}{1}\right)$$

(c) $\left(\dfrac{f}{g}\right)(1) = \dfrac{2}{3}$

$$= -6$$

(d) $\left(\dfrac{f}{g}\right)(1) = -6$