January 23 MATH 1112 sec. 54 Spring 2019

Section 2.1: More on Functions

In this section we look at two concepts:

- What does it mean for a function to be increasing, decreasing or constant on an interval? How would this affect the graph of the function.
- What are piece-wise defined functions? How are they evaluated, and how are they graphed?

Graphing Functions: Increasing, Decreasing

Some definitions:

Suppose that the function f is defined on an open interval I.

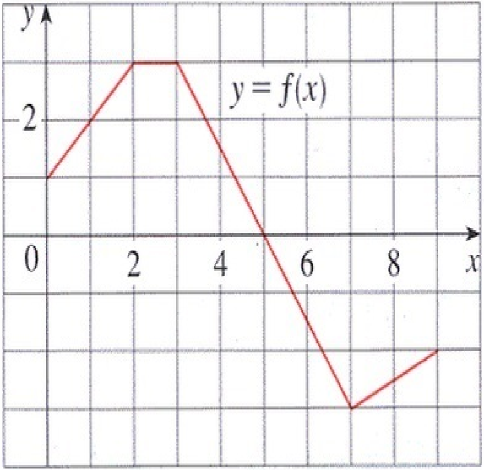
- f is *increasing* on I if for each a, b in I, if $a < b$, then $f(a) < f(b)$.
- f is *decreasing* on I if for each a, b in I, if $a < b$, then $f(a) > f(b)$.
- f is *constant* on I if $f(a) = f(b)$ for each a, b in I.

Note that going from left to right, the graph of f

- goes upward if f is increasing
- goes downward if f is decreasing
- is horizontal if f is constant.

Example

Identify the intervals (if any) on which f is increasing.



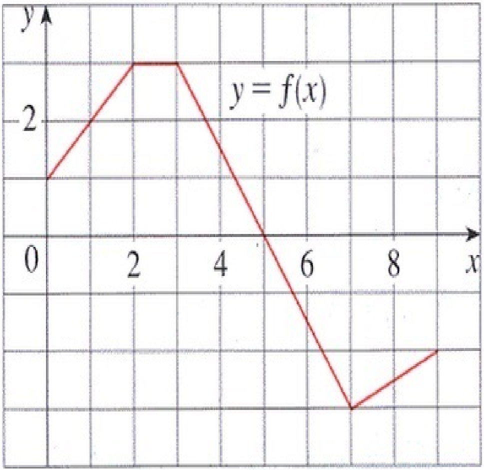
going up
from left to right

(0,2) is one interval

(7,9) is the other interval

Example

Identify the intervals (if any) on which f is decreasin g and any intervals on which f is constant.



f is decreasing on

$(3, 7)$

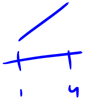
f is constant on

$(2, 3)$.

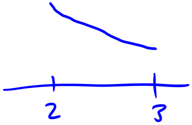
Question

Suppose the function $g(x)$ is **decreasing** on the interval $(0, 7)$. Which of the following is true?

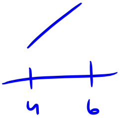
(a) $g(1) < g(4)$



(b) $g(2) > g(3)$



(c) $g(4) < g(6)$



(d) All of the above are true.

(e) None of the above are true.

Relative Extrema

Some defintions:
Suppose f is a function and c is in the interior of the domain of f. Then

- $f(c)$ is a **relative maximum** is there exists an open interval I containing c such that $f(x) < f(c)$ for all x in I different from c,
- $f(c)$ is a **relative minimum** is there exists an open interval I containing c such that $f(x) > f(c)$ for all x in I different from c.

An **extremum** is a maximum or a minimum. The plurals of these three terms are extrema, maxima, and minima. The word *relative* can be replaced with the word **local**.

Relative Extrema

Relative extrema are the y-values for local highest or lowest points on a graph.

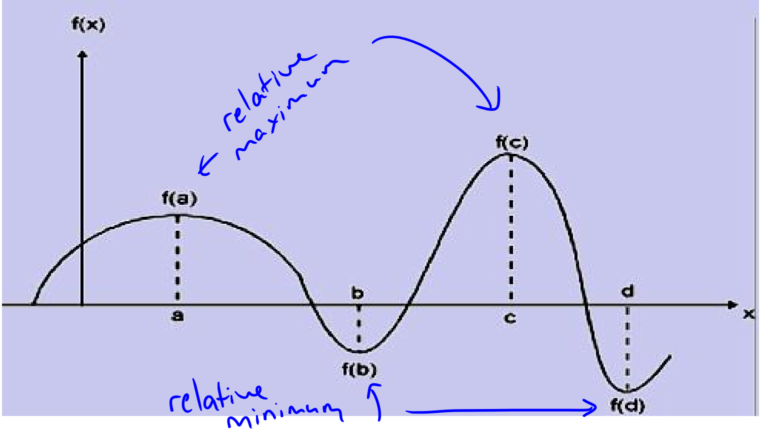


Figure: f has relative maxima $f(a)$ and $f(c)$ and relative minima $f(b)$ and $f(d)$

Evaluating and Graphing Piecewise Defined Functions

We wish to consider functions that are defined by different rules over different parts of the domain. These are called piecewise defined functions. An example is

$$f(x) = \begin{cases} x^2 - 1, & x < 0 \\ 2, & 0 \le x < 1 \\ \frac{1}{x}, & x \ge 1 \end{cases}$$

Example Evaluating Piecewise Defined Functions

Let $$f(x) = \begin{cases} x^2 - 1, & x < 0 \\ 2, & 0 \le x < 1 \\ \frac{1}{x}, & x \ge 1 \end{cases}$$

Evaluate

(a) $f(4) = \frac{1}{4}$ $\qquad 4 \geq 1$ rule 3

(b) $f(-\pi) = (-\pi)^2 - 1 = \pi^2 - 1$ $\qquad -\pi < 0$

(c) $f(0) = 2$ $\qquad 0 \leq 0 < 1$

(d) $f\left(\frac{1}{3}\right) = 2$ $\qquad 0 \leq \frac{1}{3} < 1$

Question

Evaluate $g(2)$ where

$$g(x) = \begin{cases} \frac{x+1}{x-2}, & x \leq -3 \\ x^3 - 2x, & -3 < x < 1 \\ \frac{x}{2} - 3, & x \geq 1 \end{cases}$$

(a) $g(2)$ is undefined because of division by zero

(b) $g(2) = 4$

(c) $g(2) = -2$

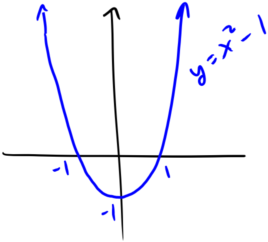
$2 \geq 1$ $\quad g(2) = \frac{2}{2} - 3 = 1 - 3 = -2$

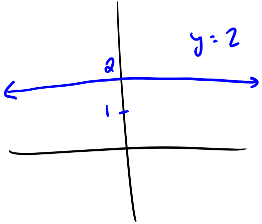
(d) $g(2)$ is undefined because 2 is not in the domain

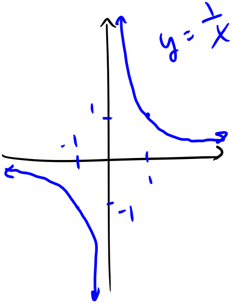
Plotting Piecewise Defined Functions

If we know how to plot the different pieces of a piecewise defined function, then we can sketch its plot. As an example, let's plot

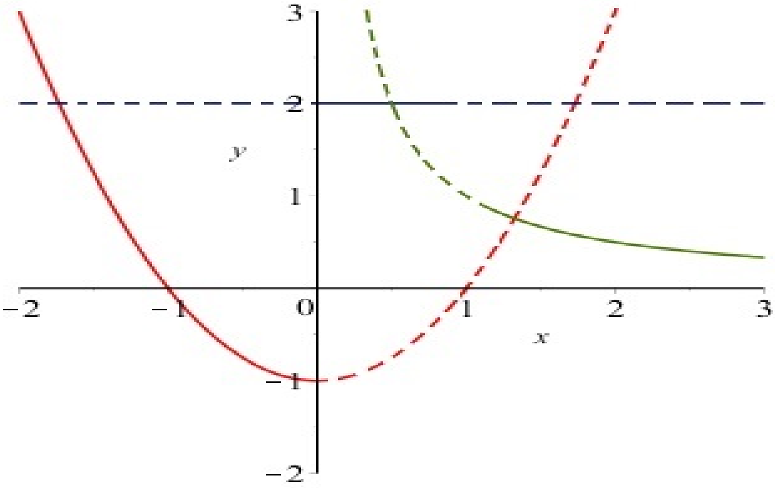
$$f(x) = \begin{cases} x^2 - 1, & x < 0 \\ 2, & 0 \le x < 1 \\ \frac{1}{x}, & x \ge 1 \end{cases}$$





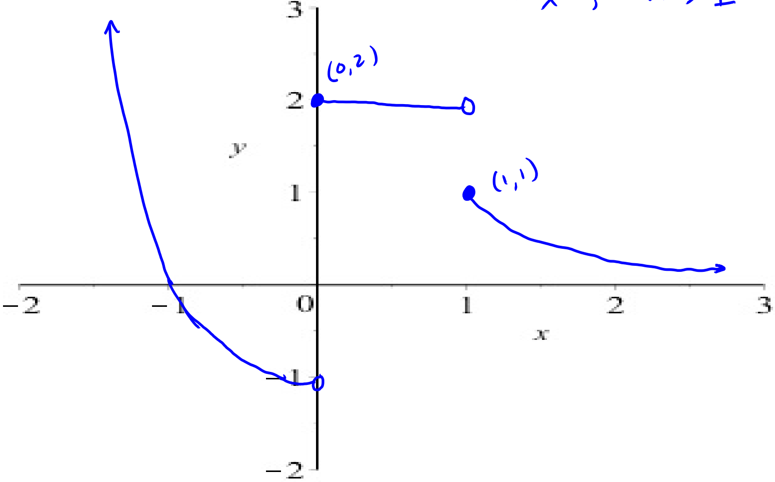


Plotting Example



Plotting Example

$$f(x) = \begin{cases} x^2 - 1, & x < 0 \\ 2, & 0 \le x < 1 \\ \frac{1}{x}, & x \geq 1 \end{cases}$$



Piecewise Defined Functions

Let $f(x) = \begin{cases} \frac{x}{x-2}, & x < 2 \\ 1, & x = 2, \\ x^2, & x > 2 \end{cases}$. Suppose that $h > 0$, and evaluate

(a) $f(2) = 1$ (second rule)

(b) $f(2+h) = (2+h)^2 = 4+4h+h^2$ $\quad$ $2+h>2$ since $h>0$

(c) $f(2-h) = \dfrac{2-h}{2-h-2} = \dfrac{2-h}{-h} = -\dfrac{2-h}{h}$ $\quad$ $2-h<2$ since $h>0$