January 25 Math 2306 sec. 57 Spring 2018

Section 4: First Order Equations: Linear

We will find the general solution of the first order linear equation in **standard form**

$$\frac{dy}{dx} + P(x)y = f(x).$$

We assume that P and f are continuous on the domain of definition. The general solution (a 1-parameter family) will have the form

$$y = y_c + y_p$$

where y_c is the complementary solution to the associated homogeneous equation (y' + Py = 0) and y_p is called a particular solution.

Derivation of Solution via Integrating Factor

Solve the equation in standard form

$$\frac{dy}{dx} + P(x)y = f(x)$$

we will multiply by a certain function p(x) such that the resulting left side will be a single derivative, product rule term. It will be

Melt. by p

$$\frac{d}{dx} \left[\mu y \right] = \mu \frac{dy}{dx} + \frac{d\mu}{dx} y$$

Equality of the 2nd terms requires

Concel the y's, and assume $\mu(x) > 0$.

$$\frac{d\mu}{dx} = \mu P(x)$$
 | 1st order separable !

$$\int_{\Gamma} d\mu = \int_{\Gamma} P(x) dx \Rightarrow \int_{\Gamma} \mu = \int_{\Gamma} P(x) dx$$

$$\mu = \int_{\Gamma} P(x) dx \Rightarrow \int_{\Gamma} \mu = \int_{\Gamma} P(x) dx$$

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Now we solve the ODE. mult. by properties of the Piny = prf(x)

Surankel product + printy = prf(x)

by

choice

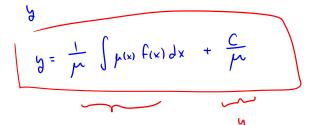
Choice

of pr

$$\int \frac{dx}{dx} \left[\mu y \right] dx = \int \mu(x) f(x) dx$$

$$\mu_{\delta} = \int \mu(x) f(x) dx + C$$

Isolate



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In terms of the exponential of P

- spordx

- spordx

- spordx

- spordx

- spordx

- spordx

General Solution of First Order Linear ODE

- ▶ Put the equation in standard form y' + P(x)y = f(x), and correctly identify the function P(x).
- ▶ Obtain the integrating factor $\mu(x) = \exp(\int P(x) dx)$.
- Multiply both sides of the equation (in standard form) by the integrating factor μ . The left hand side **will always** collapse into the derivative of a product

$$\frac{d}{dx}[\mu(x)y] = \mu(x)f(x).$$

Integrate both sides, and solve for y.

$$y(x) = \frac{1}{\mu(x)} \int \mu(x) f(x) dx = e^{-\int P(x) dx} \left(\int e^{\int P(x) dx} f(x) dx + C \right)$$



Solve the ODE

Get
$$\mu = e^{-x}$$
 It's in standard form. $P(x) = 1$

Get $\mu = e^{-x}$ $= e^{x$

$$\frac{d}{dx} \left[e \right] = 3x$$

Integrate

$$\left[\frac{1}{2}\left(\frac{1}{6}x\right)\right]dx = \int 3x dx$$

$$y = \frac{32x^2 + C}{e^x} = \frac{3}{2}x^2e^x + Ce^x$$

Solve the IVP

$$x\frac{dy}{dx} - y = 2x^{2}, x > 0 \quad y(1) = 5$$

$$\frac{dy}{dx} - \frac{1}{x} y = \frac{2x^{2}}{x} = 2x \quad \Rightarrow \quad \frac{dy}{dx} + \left(\frac{1}{x}\right)y = 2x$$

$$P(y) = \frac{1}{x} \quad \mu = e \quad = e \quad = e$$

$$= e^{\int_{\mathbf{n}} x^{1}} = x^{1} \qquad \mu = x^{1}$$

$$x^{1} \frac{dy}{dx} - x^{1} \cdot \frac{1}{x} y = x^{1}(2x)$$

= [x'5] = 2



$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left[x^{2} \partial \right] dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2 dx$$

$$= 2x + C$$

$$y = \frac{2x + C}{x^{-1}} = 2x^{2} + Cx$$

The 1-parenter family of solutions to
the ODE is
$$y = 2x^2 + Cx$$
.

To solve the IVP, we apply &(1)=5.

Verify

Just for giggles, lets verify that our solution $y = 2x^2 + 3x$ really does solve the differential equation we started with

$$x \frac{dy}{dx} - y = 2x^{2}.$$
If $y = 2x^{2} + 3x$, then $\frac{dy}{dx} = 4x + 3$

$$= 4x^{2} + 3x - 2x^{2} - 3x$$

$$= 4x^{2} + 3x - 2x^{2} - 3x$$

$$= 4x^{2} - 2x^{2} + 3x - 3x$$

$$= 3x^{2}$$

Steady and Transient States

For some linear equations, the term y_c decays as x (or t) grows. For example

$$rac{dy}{dx}+y=3xe^{-x}$$
 has solution $y=rac{3}{2}x^2e^{-x}+Ce^{-x}.$ Here, $y_p=rac{3}{2}x^2e^{-x}$ and $y_c=Ce^{-x}.$

Such a decaying complementary solution is called a **transient state**.

The corresponding particular solution is called a steady state.

