January 27 MATH 1112 sec. 54 Spring 2020

Quick Review of Inverse Functions

Suppose we have the function $y = f(x)$ where $f(x) = x^3 - 1$. We can say

when the input $x = 2$, the output $y = 7$.

We can also say it the other way around

when the output $y = 7$, the input $x = 2$.

It doesn't always work so nicely. Consider the example

$$f(x) = x^2.$$

When $x = 2$, we have $f(x) = 4$. But if $f(x) = 4$, we can't be certain about what x is!

One to One

Definition: A function f is **one to one** if different inputs have different outputs. That is f is one to one provided

$$a \neq b \quad \text{implies} \quad f(a) \neq f(b).$$

Equivalently, f is a one to one function provided

$$f(a) = f(b) \quad \text{implies} \quad a = b.$$

Horizontal Line Test: A function $f(x)$ is one to one if and only if the graph of $y = f(x)$ is intersected at most one time by every horizontal line.

Horizontal Line Test

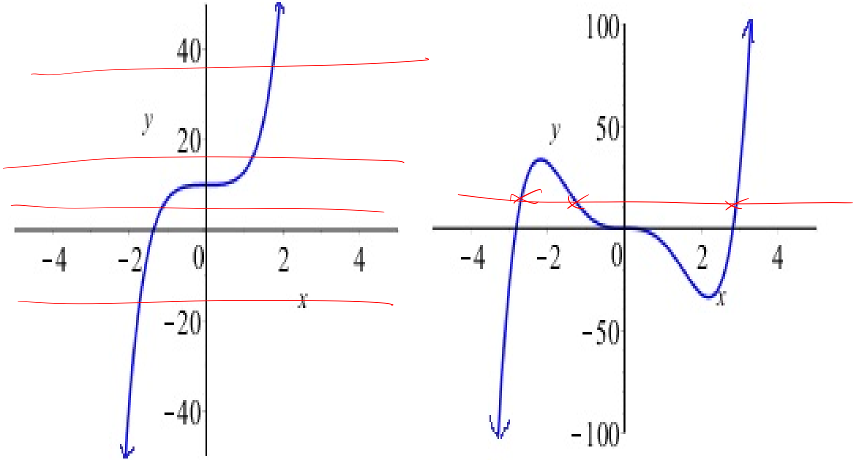
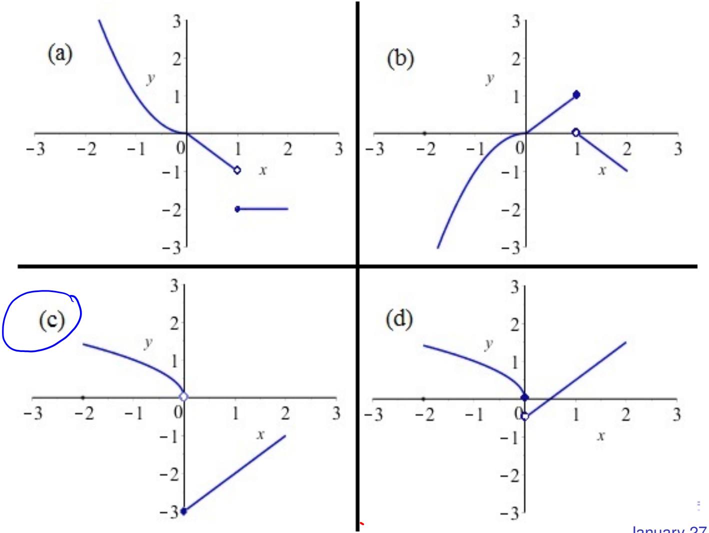


Figure: Left: A one to one function. Right: A function that is not one to one.

Question

Which of the following is the graph of a one to one function? (Hint: Horizontal Line Test)



Inverse Function

Theorem: If f is a one to one function with domain D and range R, then its inverse f^{-1} is a function with domain R and range D. Moreover, the inverse function is defined by

$$f^{-1}(x) = y \quad \text{if and only if} \quad f(y) = x.$$

Note that inverse functions *swap* domains and ranges, inputs and outputs.

Question

Suppose f is a one-to-one function.

If the graph of $y = f(x)$ passes through the points $(2, 4)$, then which of the following **must** be true?

(a) $f^{-1}(2) = 4$

(b) $f^{-1}\left(\frac{1}{2}\right) = \frac{1}{4}$

(c) $f^{-1}(4) = 2$

(d) $f^{-1}\left(\frac{1}{4}\right) = \frac{1}{2}$

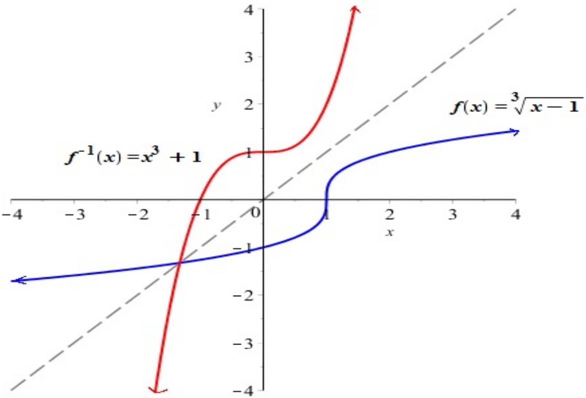
Characteristic Compositions:

If f is a one to one function with domain D, range R, and with inverse function f^{-1}, then

- for each x in D, $(f^{-1} \circ f)(x) = x$, and
- for each x in R, $(f \circ f^{-1})(x) = x$.

Graphs

If (a, b) is a point on the graph of a function, then (b, a) is a point on the graph of its inverse. So the graph of f^{-1} is obtained by reflecting the graph of f in the line $y = x$.



Restricting the Domain

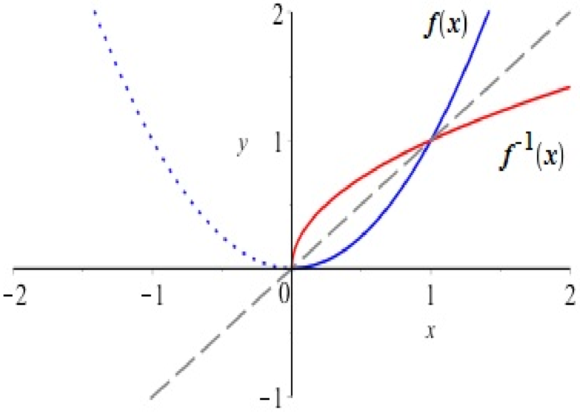


Figure: If the domain of $y = x^2$ is restricted to $[0, \infty)$, the graph passes the horizontal line test. $f(x) = x^2$ for $x \geq 0$ has inverse function $f^{-1}(x) = \sqrt{x}$.