January 30 MATH 1112 sec. 54 Spring 2019

Section 2.4: Symmetry

Definition: If a function f is called an **even function** if

$$f(-x) = f(x)$$

for each x in its domain. We can say that such a function has **even symmetry**.

Definition: If a function f is called an **odd function** if

$$f(-x) = -f(x)$$

for each x in its domain. We can say that such a function has **odd symmetry**.

Even Symmetry

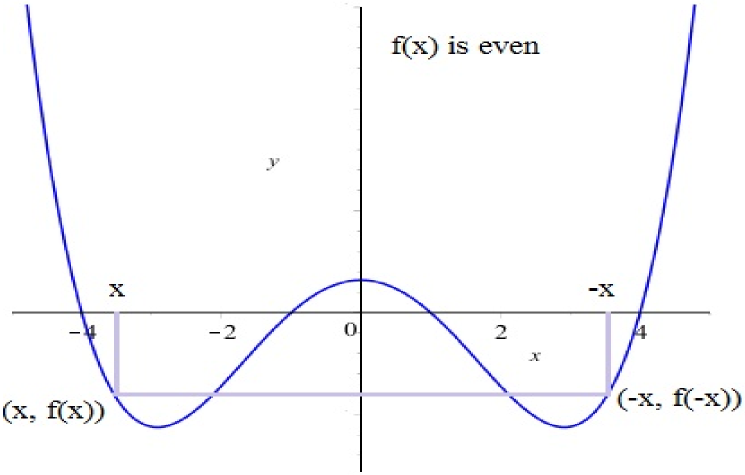


Figure: The graph to the left of the *y*-axis is the mirror image of the graph on the right side if a function is even.

Odd Symmetry

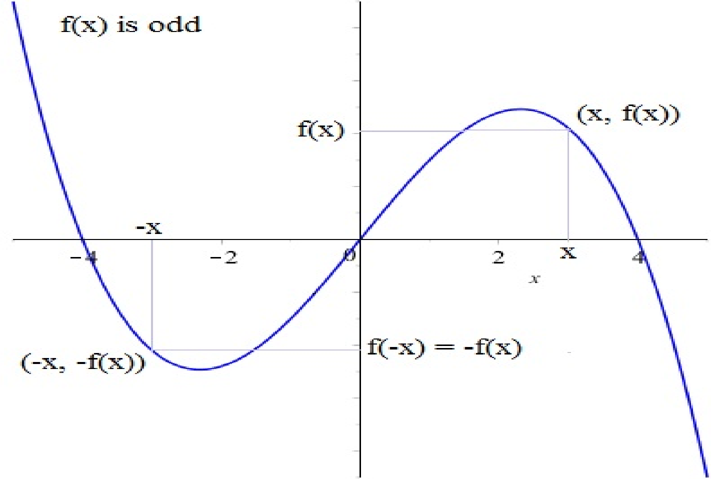


Figure: The graph of f to the left of the y-axis can be obtained by reflecting the graph on the right twice—through the y-axis and then the x-axis.

Even and Odd Symmetry

- Polynomials with only even powers (including 0) are even functions. Polynomials with only odd powers are odd functions.
- Even symmetry is called symmetry with respect to the y-axis.
- Odd symmetry is called symmetry with respect to the origin.
- Not all functions have symmetry (for example polynomials with both even and odd power terms). Some important functions have known symmetry.

Question

Let $f(x) = |x| - 2x^2$. Then $f(-x) =$ $|-x| - 2(-x)^2$

$= |x| - 2x^2$

(a) $-|x| + 2x^2$

(b) $|x| + 2x^2$

(c) $|x| - 2x^2$

(d) $|x| + 2x^2$

Question

Let $f(x) = |x| - 2x^2,$ then

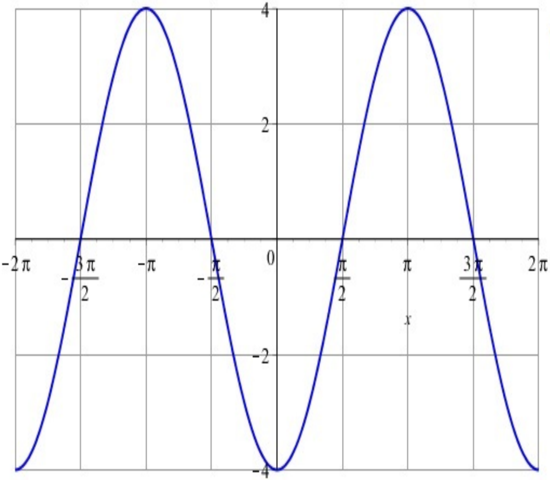
(a) f is an odd function.

(b) f is an even function.

(c) f is both an even and an odd function.

(d) f is neither even nor odd.

Question



The figure shows the plot of a function $y = f(x)$ that is

(a) Even

(b) Odd

(c) Both Even and Odd

(d) Neither Even nor Odd

x-axis Symmetry

Consider the relation $\sqrt[3]{(x-1)^2} + \sqrt[3]{y^2} = 1$. Note that if we replace y with $-y$ on the left side, we get

$$\sqrt[3]{(x-1)^2} + \sqrt[3]{(-y)^2} = \sqrt[3]{(x-1)^2} + \sqrt[3]{y^2}$$

So if (x, y) is on the graph of the relation, so is $(x, -y)$. Such a function is said to have **symmetry with respect to the x-axis**—or just x-axis symmetry for short.

x-axis Symmetry

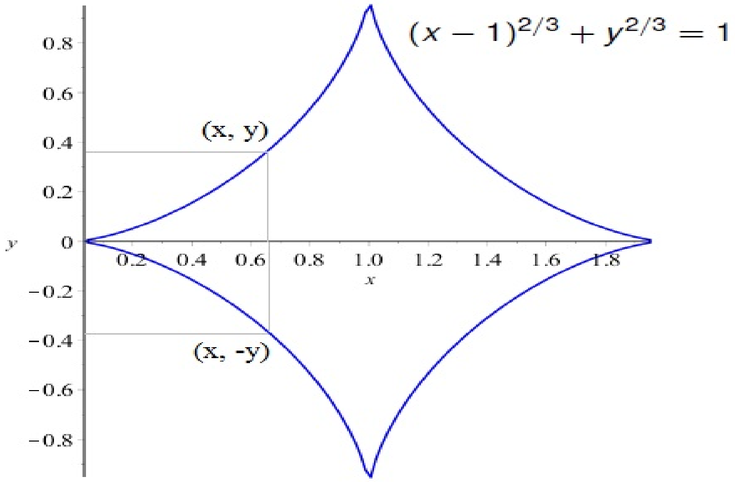


Figure: The part of the graph below the x-axis is the mirror image of the part above the x-axis.

Symmetry Checks

For a function or a relation given in terms of algebraic expressions, we can check for symmetry:

- **Even:** if replacing (x, y) with $(-x, y)$ results in the same formula (i.e. $f(-x) = f(x)$)
- **Odd:** if replacing (x, y) with $(-x, -y)$ results in the same foruma (i.e. $f(-x) = -f(x)$)
- **x-axis:** if replacing (x, y) with $(x, -y)$ results in the same formula.

Question

Jack and Diane are working together to graph a function. Jack thinks they should test the **function** for *x*-axis symmetry. Diane says it's not necessary to check the **function** for *x*-axis symmetry. Who is correct, and why?

(a) Jack is correct because all functions have *x*-axis symmetry.

(b) Jack is correct because some but not all functions have *x*-axis symmetry.

(c) Diane is correct because very few functions have *x*-axis symmetry.

(d) Diane is correct because no function can have *x*-axis symmetry.

Think "vertical line test"