January 6 Math 3260 sec. 55 Spring 2020

A Random Motivational Example

In a certain city, ABC shipping has one receiving (A) and two distribution hubs (B & C). On a given day, 80 packages enter center A and will be distributed to hubs B and C for delivery. Twenty packages will go to a major client from hub C, the rest are to be distributed in quantities $x_1, \ldots, x_4$ among the hubs and out for delivery.

Motivating Example

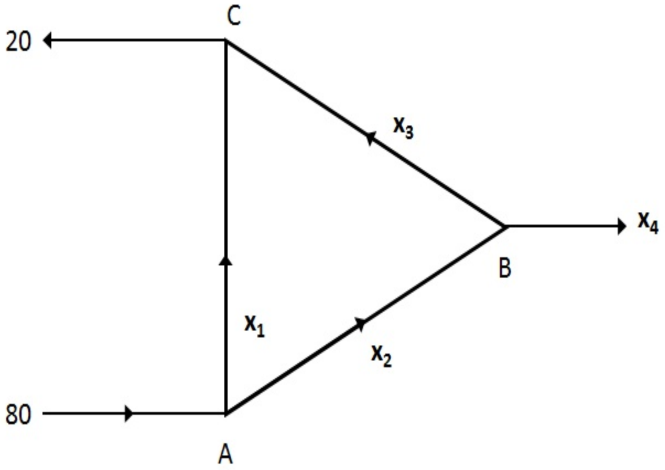

Figure: Distribution Scheme

Equations for Package Quantities

Assuming all of the packages are delivered to customers outside of the shipping company, the quantities $x_1, \ldots, x_4$ have to satisfy the equations

$$\begin{array}{ccccccccc} x_1 & & & + & x_3 & & & = & 20 \\ & & x_2 & - & x_3 & - & x_4 & = & 0 \\ x_1 & + & x_2 & & & & & = & 80 \end{array}$$

Questions

- Is there a set of numbers $x_1, \ldots, x_4$ that satisfy all of the equations?
- If there is a set of numbers, is it the only one?
- If we could find numbers $x_1, \ldots, x_4$, and then the input 80 changed (say on another day), do we have to do all the work again? Or is there a way to generalize our finding?

(This is just to illustrate the kinds of questions addressed by **Linear Algebra.** We'll leave answering these questions for another day.)

Section 1.1: Systems of Linear Equations

We begin with a linear (*algebraic*) equation in n variables x_1, x_2, ..., x_n for some positive integer n.

A **linear equation** can be written in the form

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b.$$

The numbers $a_1, \ldots, a_n$ are called the *coefficients*. These numbers and the right side b are real (or complex) constants that are **known**.

Examples of Equations that are or are not Linear

$$2x_1 = 4x_2 - 3x_3 + 5 \quad \text{and} \quad 12 - \sqrt{3}(x + y) = 0$$

$2x_1 - 4x_2 + 3x_3 = 5$ $\qquad$ $\sqrt{3}\,x + \sqrt{3}\,y = 12$

$$x_1 + 3x_3 = \frac{1}{x_2} \quad \text{and} \quad xyz = \sqrt{w}$$

non linear term $\qquad$ both nonlinear

A *Linear System* is a collection of linear equations in the same variables

$$\begin{array}{rcrcrcrcr} 2x_1 & + & x_2 & - & 3x_3 & + & x_4 & = & -3 \\ -x_1 & + & 3x_2 & + & 4x_3 & - & 2x_4 & = & 8 \end{array}$$

$$\begin{array}{rcrcrcr} x & + & 2y & + & 3z & = & 4 \\ 3x & & & + & 12z & = & 0 \\ 2x & + & 2y & - & 5z & = & -6 \end{array}$$

Some terms

- A **solution** is a list of numbers $(s_1, s_2, \ldots, s_n)$ that reduce each equation in the system to a true statement upon substitution.
- A **solutions set** is the set of all possible solutions of a linear system.
- Two systems are called **equivalent** if they have the same solution set.

An Example

$$\begin{array}{rcrcr} 2x & - & y & = & -1 \\ -4x & + & 2y & = & 2 \end{array}$$

(a) Show that $(1, 3)$ is a solution.

We'll substitute $x = 1$ and $y = 3$

1st equation $2(1) - (3) \stackrel{?}{=} -1$

$2 - 3 \stackrel{?}{=} -1$

$-1 = -1$ an identity

2nd equation $-4(1) + 2(3) \stackrel{?}{=} 2$

$-4 + 6 \stackrel{?}{=} 2$

$2 = 2$ an identity!

So $(1,3)$ is a solution.

$x = 1, \ y = 3$

An Example Continued

$$\begin{array}{rcrcr} 2x & - & y & = & -1 \\ -4x & + & 2y & = & 2 \end{array}$$

(b) Note that $\{(x, y) | y = 2x + 1\}$ is the solution set.

Using substitution of $y = 2x + 1$

1st equation

$$2x - (2x + 1) \stackrel{?}{=} -1$$

$$2x - 2x - 1 \stackrel{?}{=} -1$$

$$-1 = -1$$

an identity

2nd equation

$$-4x + 2(2x + 1) \stackrel{?}{=} 2$$

$$-4x + 4x + 2 \stackrel{?}{=} 2$$

$2 = 2$ an identity.

So all (x, y) such that $y = 2x + 1$ is a solution.

The Geometry of 2 Equations with 2 Variables

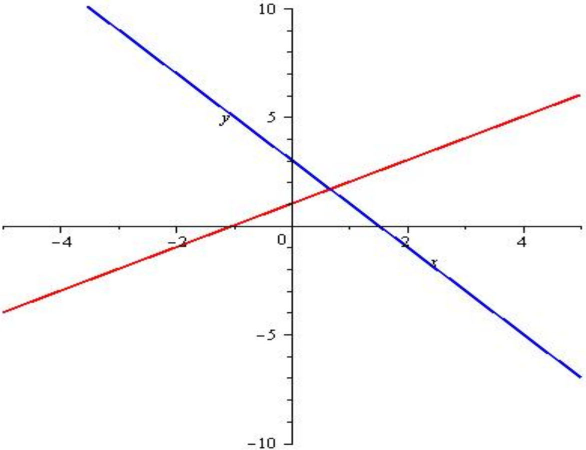

Figure: The system $x - y = -1$ and $2x + y = 3$ with solution set $\{(2/3, 5/2)\}$. These equations represent lines that intersect at one point.

The Geometry of 2 Equations with 2 Variables

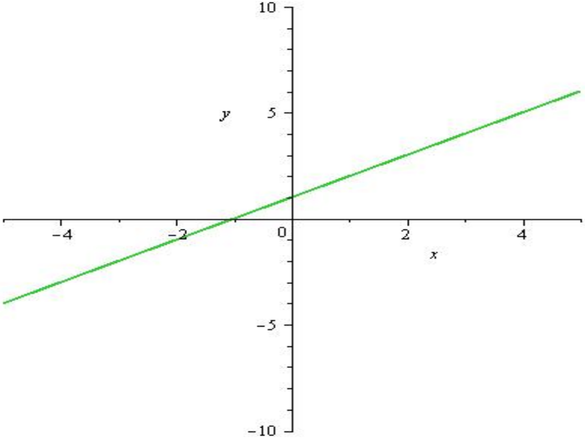

Figure: The system $x - y = -1$ and $2x - 2y = -2$ with solution set $\{(x, y)|y = x + 1\}$. Both equations represent the same line which share all common points as solutions.

The Geometry of 2 Equations with 2 Variables

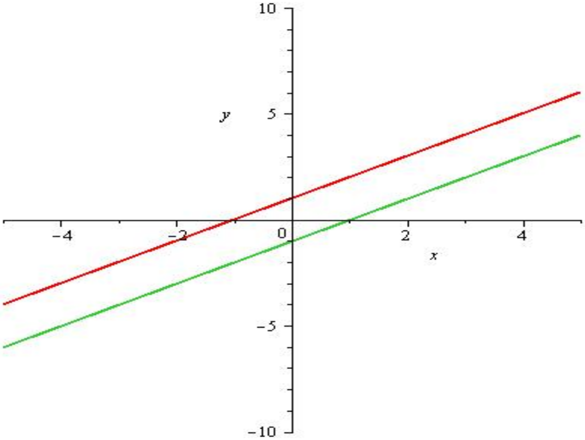

Figure: The system $x - y = -1$ and $2x - 2y = 2$ with solution set $\emptyset$. These equations represent parallel lines having no common points.