January 8 MATH 1112 sec. 54 Spring 2020

Relations & Functions

Given two sets of objects, we can define a **relation** on these sets.

Definition: A **relation** is a correspondence between a first set, called the **domain**, and a second set, called the **range**.

We can call a relation a **mapping** as it assigns (maps) one or more range elements to a domain element.

Examples: Relation in x and y

A **relation** is a set of **ordered** pairs, (x, y). The set of x values is the **domain**, and the set of y values is the **range.**

Example 1: Identify the domain and range of

$$\{(0,0),(1,-1),(1,1),(4,2)\}$$

Domain: Set of 1^{st} elements

$\{0, 1, 4\}$

Range: Set of 2^{nd} elements

$\{-1, 0, 1, 2\}$

Examples: Relation in x and y

Example 2: Identify the domain and range of

$$\{(-1,1),(0,0),(1,1),(2,4)\}$$

Domain: $\{-1, 0, 1, 2\}$

Range: $\{0, 1, 4\}$

Notice that no two ordered pairs in this relation have the same x-value. This is an example of a **function**.

Definition: A **function** on two sets is a relation in which each element of the domain corresponds to exactly **one** element in the range.

Question $\longleftarrow$ (indicates a clicker poll)

A **function** is a relation.

(a) True

(b) False

(c) I think it's true, but I'm not sure.

(d) I think it's false, but I'm not sure.

Question

Which (if any) of the following relations is a function?

(a) $\{(0,0),(1,1),(2,2)\}$

(b) $\{(0,1),(0,2),(0,3)\}$

(c) $\{(1,3),(-1,3),(7,7),(0,7)\}$

(d) (a) and (c)

(e) none of the above

Function Notation: Some Preliminary Remarks

- We will use a variable character to represent domain elements—usually x (but not always).

- We will use a variable character to represent corresponding range elements—usually y (again, we're not married to y).

- We assign a character name to our functions as well using specific notation—often we use f, sometime g, h or something else.

Function Notation: Some Preliminary Remarks

- The domain and range are not always stated explicitly, but we can often infer them.

- **Reading, writing, and using function notation properly is one of the most critical skills focused on in this class.**

Function Notation: An example

Consider the equation $y = 3x - 4$. It defines a set of points

$$(x, y) = (x, 3x - 4)$$

where x and y are elements of the set of real[1] numbers $\mathbb{R}$.

The equation $y = 3x - 4$ defines a function. Let's call this function f. We can express this in **function notation** as

$$f(x) = 3x - 4.$$

In English, this reads as

f of *x* equals three *x* minus 4.

[1] The symbol $\mathbb{R}$ denotes the set of all real number.

Function Notation: An example

$$\text{Let} \quad f(x) = 3x - 4, \quad \text{and suppose} \quad y = f(x)$$

- In $f(x)$, f is the function and x is its **argument**.
- x represents an element of the domain, $f(x)$ is an element of the range.
- Since $y = f(x)$, x is called the **independent variable** and y is called the **dependent variable**.
- $y = f(x)$ reads "y equals f of x"
- The collection of points $(x, f(x))$, for each x in the domain, is called **the graph of f**.

Example

Consider the function f defined by $f(x) = -x^2 + 2x + 4$. Evaluate each of the following.

(a) $f(-2) = -(-2)^2 + 2(-2) + 4 = -4 - 4 + 4 = -4$

$* \quad -2^2 = -4$ and $(-2)^2 = 4$

(b) $f(3a) = -(3a)^2 + 2(3a) + 4 = -9a^2 + 6a + 4$

(c) $f(x+h) = -(x+h)^2 + 2(x+h) + 4 = -(x^2 + 2xh + h^2) + 2x + 2h + 4$

$= -x^2 - 2xh - h^2 + 2x + 2h + 4$

Question

Let $f(x) = -x^2 + 2x + 4$. Evaluate $f(3)$.

(a) $f(3) = 1$

$-9 + 6 + 4 = 1$

(b) $f(3) = 7$

(c) $f(3) = -3$

(d) $f(3) = 19$

Question

Let $f(x) = -x^2 + 2x + 4$. Evaluate $f(-b)$.

(a) $f(-b) = -3b + 4$

(b) $f(-b) = -b^2 + 2b + 4$

(c) $f(-b) = b^2 - 2b + 4$

(d) $f(-b) = -b^2 - 2b + 4$

$= -(-b)^2 + 2(-b) + 4$

$= -b^2 - 2b + 4$