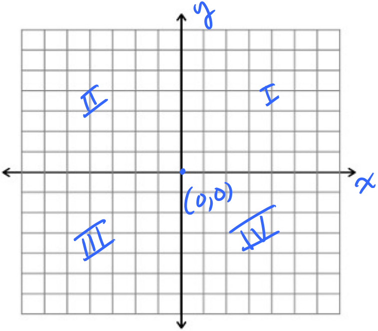
January 9 MATH 1112 sec. 54 Spring 2019

Section 1.1 (Section numbers refer to Bittinger)

Review of the Cartesian Coordinate System, Graphs, and Circles

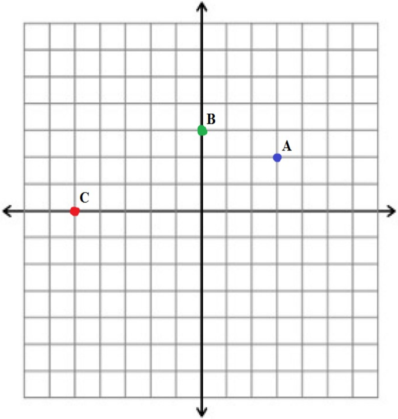


Quadrants I through IV

Figure: The x-axis and y-axis meet at the **origin**. These divide the plane into four quadrants numbered counter-clockwise starting from the upper right.

Question

Identify the coordinates of the three points A, B, and C. (Assume each square is one unit by one unit.)



(a) $A = (3, 2)$ $B = (3, 0)$, $C = (0, -5)$

(b) $A = (3, 2)$ $B = (0, 3)$, $C = (-5, 0)$

(c) $A = (2, 3)$ $B = (0, 3)$, $C = (5, 0)$

(d) $A = (3, 2)$ $B = (0, 3)$, $C = (0, -5)$

(e) can't be determined without more information

Distance and Midpoint

Theorem: The distance d between a pair of points (x_1, y_1) and (x_2, y_2) in the plane is

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Note that the order in which the points are designated does not matter.

Theorem: If a line segment has end points at the coordinates (x_1, y_1) and (x_2, y_2), then the midpoint has coordinates

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Note that the coordinates of the midpoint are the (arithmetic) averages of the corresponding end point coordinates.

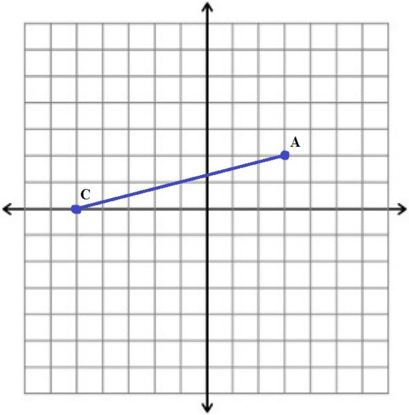
Question

Given the points A and C, determine the length d and midpoint M of the segment $\overline{AC}$.

$A = (3,2)$ $C = (-5,0)$

$d = \sqrt{(3-(-5))^2 + (2-0)^2} = \sqrt{64+4}$

$= \sqrt{68}$



(a) $d = 10, \quad M = (-1, 1)$

(b) $d = 10, \quad M = (4, 1)$

(c) $d = \sqrt{68}, \quad M = (-1, 1)$

(d) $d = \sqrt{68}, \quad M = (4, 1)$

(e) can't be determined without more information

Two Equations in Two Variables (section 9.1)

Jazz empties his pocket to find 21 coins, all nickels and pennies. He counts 49 cents. How many nickels does he have?

If we let P be the number of pennies and N the number of nickels, the above scenario can be expressed mathematically by the pair of equations

$$\begin{array}{rcrcrl} P & + & N & = & 21 & \text{(number of coins equation)} \\ 1P & + & 5N & = & 49 & \text{(number of cents equation)} \end{array}$$

Both equations are supposed to hold true, so the numbers P and N must satisfy this **system of equations.**

Example Solving a System using Substitution

$$\begin{aligned} P &+ N &= 21 \\ P &+ 5N &= 49 \end{aligned}$$

The idea is to write P in terms of N, or N in terms of P. Then substitute into the other equation to get an equation with only one variable.

From $P+N=21$, $P=21-N$.

Set P to $21-N$ in the 2^{nd} equation.

$$(21-N)+5N=49$$

Now solve for N

$$21 - N + 5N = 49$$

$$21 + 4N = 49$$

$$4N = 49 - 21 = 28$$

$$N = \frac{28}{4} = 7$$

Jazz has 7 nickels (and 14 pennies).

Example Solving a System using Elimination

$$\begin{aligned} P + N &= 21 \\ P + 5N &= 49 \end{aligned}$$

Here, we add / subtract multiples of the equations together so that one variable has zero coefficient – i.e. is eliminated.

We can subtract one equation from the other to eliminate P.

Subtract top equation from bottom one

$$\begin{aligned} & P + 5N = 49 \\ -(& P + N = 21) \\ \hline \end{aligned}$$

$$(0)\ P + (5-1)N = 49-21 = 28$$

$$4N = 28$$

$$N = 7$$

Again, Jazz has 7 Nickels

System Solution: Graphical Representation

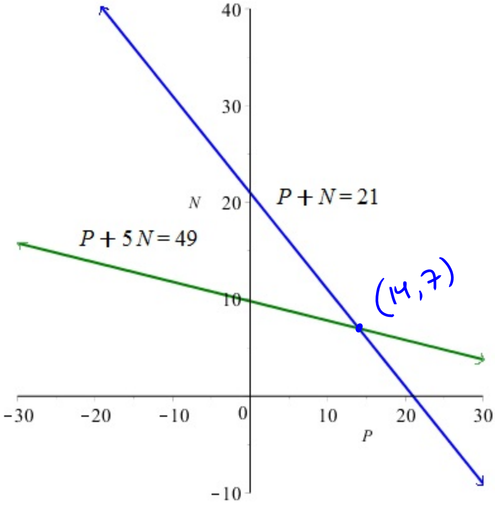


Figure: The lines $N = -P + 21$ and $N = -\frac{1}{5}P + \frac{49}{5}$ on one set of axes.

Two Equations in Two Variables

Theorem: Let a, b, c, d, e, and f be fixed constants. The system of equations

$$\begin{array}{rcrcl} ax & + & by & = & e \\ cx & + & dy & = & f \end{array}$$

satisfies one of three cases:

- It has exactly one solution.
- It has infinitely many solutions.
- It has no solutions.

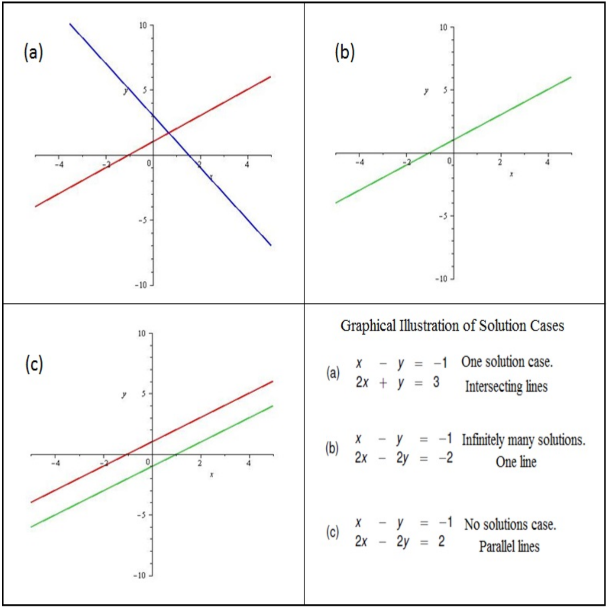
If the system has a solution (first two cases), it is called **consistent**. If it has no solutions, it is called **inconsistent**.

Consistent and Inconsistent Systems

Consistent Independent: A system is called this when it has exactly one solution. Graphically, two lines intersect in one point.

Consistent Dependent: A system is called this when it has infinitely many solutions. Graphically, the equations define the same line. All points on that line represent solutions.

Inconsistent: A system is called this when it has no solutions. Graphically, the equations define distinct, parallel lines.


(a)
(b)
(c)
Graphical Illustration of Solution Cases
(a) $x - y = -1$, $2x + y = 3$ One solution case. Intersecting lines
(b) $x - y = -1$, $2x - 2y = -2$ Infinitely many solutions. One line
(c) $x - y = -1$, $2x - 2y = 2$ No solutions case. Parallel lines

Example

Determine if the system is consistent. If so, characterize the solution.

$$\begin{array}{rcrcl} 2x & + & y & = & 3 \\ x & + & \frac{1}{2}y & = & \frac{3}{2} \end{array}$$

Question

Consider the system of equations

$$\begin{array}{rcrcl} x & - & y & = & 3 \\ 3x & + & y & = & 5 \end{array}$$

(a) This is consistent, independent with solutions $(4, 1)$.

(a) This is consistent, independent with solutions $(2, -1)$.

(a) This is consistent, dependent with infinitely many solutions.

(a) This is inconsistent.

Example

Determine if the system is consistent. If so, characterize the solution.

$$\begin{array}{rcrcr} 2x & + & y & = & 3 \\ x & + & \frac{1}{2}y & = & -4 \end{array}$$

Circles (Back to Section 1.1)

Definition: A **circle** is the set of all points in a plane equidistant from a fixed point called the center. The fixed distance is called the **radius.**

Equation of a circle: If the point (h, k) is the center of a circle of radius r in the Cartesian plane, then the set of points (x, y) on the circle satisfy

$$(x-h)^2+(y-k)^2=r^2.$$

The above is refered to as the **standard form** of the equation of the circle.

Question

The equation

$$(x-2)^2 + y^2 = 5$$

defines a circle with

(a) center $(-2, 0)$ and radius 5

(b) center $(0, 2)$ and radius $\sqrt{5}$

(c) center $(2, 0)$ and radius $\sqrt{5}$

(d) center $(2, 0)$ and radius 25

Example

Plot the circle whose points (x, y) satisfy the equation

$$x^2 + y^2 - 2x + 4y - 4 = 0$$