July 19 Math 1190 sec. 51 Summer 2017

A plane is flying at a fixed altitude and is tracked by a radar. When the horizontal distance between the plane and the radar station is 10 mi, the pilot notes a flying speed of 400 mi/hr. At this same instant, the radar attendant observes that the straight line distance between the plane and the radar is changing at a rate of 320 mi/hr. What is the altitude of the plane?

Question

What *type* of problem is this?

(a) an applied optimization problem

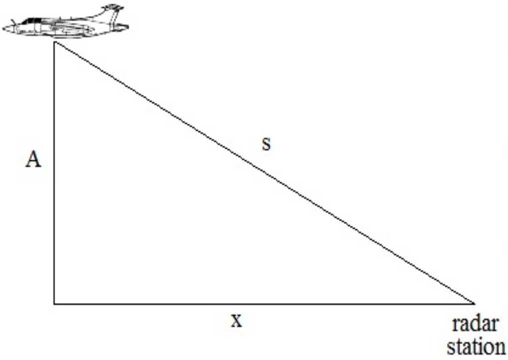
(b) an area finding problem

(c) a related rates problem

(d) a Newton's law of cooling problem

Question

Letting x be the plane's distance from the station, A the altitude, and s the straight line distance, x, A, and s are related by



(a) $A^2 = x^2 + s^2$

(b) $x^2 = A^2 + s^2$

(c) $s^2 = x^2 + A^2$

(d) $s^2 = x^2 + A^2 - 2xA\cos 30^\circ$

Plane Example Continued

We're given that $\frac{dx}{dt} = 400$ mi/hr and $\frac{ds}{dt} = 320$ mi/hr when $x = 10$ mi. Determine the (constant) altitude of the plane.

Example

Find the point (a,b) on the parabola $y=\sqrt{x}$ that is closest to the point $\left(\frac{9}{2},0\right)$.

Then show that the line through the points (a,b) and $\left(\frac{9}{2},0\right)$ is perpedicular to the line tangent to the graph of $y=\sqrt{x}$ at the point (a,b).

Example

Evaluate $$\frac{d}{dx}\left[\int_{\sin x}^{x^3} t^2 e^t \, dt\right]$$

Leibniz Rule

The Fundamental Theorem can be combined with the chain rule in the following general way due to Leibniz:

Suppose f is continuous on an interval I and that the functions $a(x)$ and $b(x)$ are differentiable functions with ranges contained in I. Then

$$\frac{d}{dx}\int_{a(x)}^{b(x)} f(t)\,dt = f(b(x))\,b'(x) - f(a(x))\,a'(x)$$

Question

$$\frac{d}{dx}\left[\int_{3x}^{e^x} \sin\left(\frac{1}{t}\right)\, dt\right] =$$

(a) $\sin\left(\frac{1}{e^x}\right) - \sin\left(\frac{1}{3x}\right)$

(b) $e^x \sin\left(\frac{1}{e^x}\right) - 3\sin\left(\frac{1}{3x}\right)$

(c) $-e^x \cos\left(\frac{1}{e^x}\right) + 3\cos\left(\frac{1}{3x}\right)$

(d) $e^x \sin(\ln(e^x)) - 3\sin(\ln(3x))$

Example

Find the most general antiderivative of the function
$f(x) = \dfrac{x^3 - \sqrt{x} + 3}{x}$.

Question

The most general antiderivative of $g(x) = \frac{4-5x+x^6}{x^2}$ is

(a) $G(x) = -\frac{4}{x} - 5\ln|x| + \frac{x^5}{5} + C$

(b) $G(x) = 4\ln|x^2| - 5\ln|x| + \frac{3}{7}\frac{x^7}{x^3} + C$

(c) $G(x) = -\frac{4}{3x^3} - 5\ln|x| + \frac{x^5}{5} + C$

(d) $G(x) = \frac{4x - \frac{5}{2}x^2 + \frac{x^7}{7}}{\frac{x^3}{3}} + C$

Question

True or False If a and b are any positive numbers, then

$$\int_a^b \frac{1}{x}\,dx = \ln(b) - \ln(a).$$

Question

True or False If a and b are any positive numbers, then

$$\int_a^b \frac{1}{x^2}\,dx = \ln(b^2) - \ln(a^2).$$

Example:

(a) Evaluate $\frac{d}{dx}\ln(x^2+1)$

(b) Evaluate $\int_0^2 \frac{2x}{x^2+1}\,dx$

Question

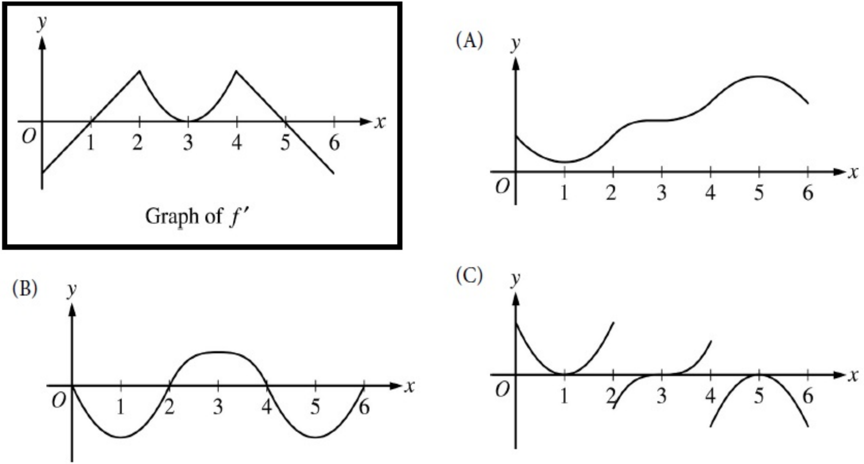


Figure: The graph of $f'(x)$ is shown in the upper left. Which of the three graphs could be the graph of $f(x)$?

Question

1. $\lim\limits_{x \to \pi} \dfrac{\cos x + \sin(2x) + 1}{x^2 - \pi^2}$ is

(A) $\dfrac{1}{2\pi}$

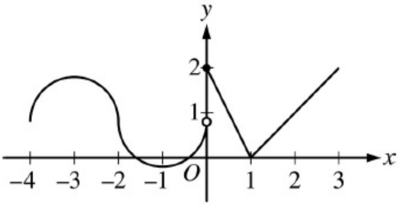
(B) $\dfrac{1}{\pi}$

(C) 1

(D) nonexistent

Figure: Evaluate the given limit.

Question



Graph of f

. The graph of the piecewise-defined function f is shown in the figure above. The graph has a vertical tangent line at $x = -2$ and horizontal tangent lines at $x = -3$ and $x = -1$. What are all values of x, $-4 < x < 3$, at which f is continuous but not differentiable?

(A) $x = 1$

(B) $x = -2$ and $x = 0$

(C) $x = -2$ and $x = 1$

(D) $x = 0$ and $x = 1$

One Sided Limits

Let f be the piecewise linear function

$$f(x) = \begin{cases} 2x - 2, & \text{for } x < 3 \\ 2x - 4, & \text{for } x \geq 3 \end{cases}$$

Compute

(I) $\displaystyle \lim_{h\to 0^-} \frac{f(3+h) - f(3)}{h}$ and

(II) $\displaystyle \lim_{h\to 0^+} \frac{f(3+h) - f(3)}{h}$

Question

Let f be the piecewise linear function

$$f(x) = \begin{cases} 2x - 2, & \text{for } x < 3 \\ 2x - 4, & \text{for } x \geq 3 \end{cases}$$

Which of the following statements is/are true

(I) $\lim_{h \to 0^-} \frac{f(3+h) - f(3)}{h} = 2$, (II) $\lim_{h \to 0^+} \frac{f(3+h) - f(3)}{h} = 2$, (III) $f'(3) = 2$

(a) none

(b) II only

(c) I and II only

(d) I and II and III

Question

If $f(x) = \int_1^{x^3} \frac{1}{1 + \ln t}\, dt$ for $x \geq 1$, then $f'(2) =$

(a) $\frac{1}{1 + \ln 2}$

(b) $\frac{12}{1 + \ln 2}$

(c) $\frac{1}{1 + \ln 8}$

(d) $\frac{12}{1 + \ln 8}$

Question

Find the derivative of the function

$$f(x) = \tan^2(\log_3 x).$$

(a) $f'(x) = \sec^4(\log_3 x)$

(b) $f'(x) = \dfrac{2\tan(\log_3 x)\sec^2(\log_3 x)}{x \ln 3}$

(c) $f'(x) = \dfrac{2}{x}\tan(\log_3 x)\sec^2(\log_3 x)$

(d) $f'(x) = \dfrac{\tan^3(\log_3 x)}{3}\dfrac{1}{x \ln 3}$

Shipping Box

A shipping company limits the size of boxes that can be shipped without additional fees. The girth[1] plus length must not exceed 108 inches. Determine the dimensions of the largest rectangular box with a square base that can be shipped without incurring extra fees.

[1] Girth is the perimeter of a cross section of the box.