June 15 Math 2254 sec 001 Summer 2015

Section 7.2: Trigonometric Integrals

$$\frac{d}{dx}\sin x = \cos x, \quad \frac{d}{dx}\cos x = -\sin x$$

$$\sin^2 x + \cos^2 x = 1$$

Half Angle IDs

$$\sin^2 x = \frac{1}{2}(1-\cos 2x), \quad \cos^2 x = \frac{1}{2}(1+\cos 2x)$$

$$\sin x \cos x = \frac{1}{2}\sin 2x$$

Other Uses of Trig Identities

Recall:

$$\frac{d}{dx}\tan x = \sec^2 x \quad \text{and} \quad \frac{d}{dx}\sec x = \sec x \tan x$$

$$\tan^2 x + 1 = \sec^2 x$$

$$\int \tan x \, dx = \ln|\sec x| + C$$

and

$$\int \sec x \, dx = \ln|\sec x + \tan x| + C$$

Other Uses of Trig Identities

Recall:

$$\frac{d}{dx}\cot x = -\csc^2 x \quad \text{and} \quad \frac{d}{dx}\csc x = -\csc x \cot x$$

$$\cot^2 x + 1 = \csc^2 x$$

$$\int \cot x\, dx = -\ln|\csc x| + C = \ln|\sin x| + C$$

and

$$\int \csc x\, dx = -\ln|\csc x + \cot x| + C$$

Evaluate

$$\int_{\pi/4}^{\pi/2} \cot^3 x \csc^4 x \, dx$$

If $u = \cot x$ then $du = -\csc^2 x \, dx$

If $u = \csc x$ then $du = -\csc x \cot x \, dx$

$$\csc^4 x \cot^3 x = \csc^3 x \cot^2 x \csc x \cot x$$

$$= \csc^3 x (\csc^2 x - 1) \csc x \cot x$$

$$= (\csc^5 x - \csc^3 x) \csc x \cot x$$

$$\int_{\pi/4}^{\pi/2} (\csc^5 x - \csc^3 x) \csc x \cot x \, dx$$

Let $u = \csc x$ $\quad du = -\csc x \cot x \, dx$

$-du = \csc x \cot x \, dx$

If $x = \pi/4$, $u = \csc \frac{\pi}{4} = \sqrt{2}$

$x = \frac{\pi}{2}$, $u = \csc \frac{\pi}{2} = 1$

$$= -\int_{\sqrt{2}}^{1} (u^5 - u^3)\, du$$

$$= -\left[\frac{u^6}{6} - \frac{u^4}{4} \right|_{\sqrt{2}}^{1}$$

$$= -\left(\frac{1^6}{6} - \frac{1^4}{4} \right) - \left[-\left(\frac{(\sqrt{2})^6}{6} - \frac{(\sqrt{2})^4}{4} \right) \right]$$

$$= -\frac{1}{6} + \frac{1}{4} + \left(\frac{8}{6} - \frac{4}{4}\right)$$

$$= \frac{7}{6} - \frac{3}{4} = \frac{7\cdot 2 - 3\cdot 3}{12} = \frac{5}{12}$$

A summary note about tangents/cotangents and secants/cosecants

- The substitution $u = \tan x$ requires we have $du = \sec^2 x\, dx$ with an even number of $\sec x$ left to convert to tangents.
- The substitution $u = \sec x$ requires we have $du = \sec x \tan x\, dx$ and an even number of $\tan x$ left to convert to secants.
- The substitution $u = \cot x$ requires we have $-du = \csc^2 x\, dx$ with an even number of $\csc x$ left to convert to cotangents.
- The substitution $u = \csc x$ requires we have $-du = \csc x \cot x\, dx$ and an even number of $\cot x$ left to convert to cosecants.

Integration by Parts w/ Trigonometric ID

Evaluate $\int \sec^3 x\, dx$ beginning with integration by parts.

$$\int \sec x \sec^2 x \, dx$$

$u = \sec x$ $\quad du = \sec x \tan x \, dx$

$v = \tan x$ $\quad dv = \sec^2 x \, dx$

$$\int \sec^3 x \, dx = \sec x \tan x - \int \sec x \tan^2 x \, dx$$

$$= \sec x \tan x - \int \sec x \left(\sec^2 x - 1\right) dx$$

$$\int \sec^3 x\,dx = \sec x \tan x - \int (\sec^3 x - \sec x)\,dx$$

$$= \sec x \tan x - \int \sec^3 x\,dx + \int \sec x\,dx$$

$$+ \int \sec^3 x\,dx \qquad\qquad + \int \sec^3 x\,dx$$

$$2\int \sec^3 x\,dx = \sec x \tan x + \int \sec x\,dx$$

$$\int \sec^3 x\,dx = \frac{1}{2}\sec x \tan x + \frac{1}{2}\int \sec x\,dx$$

$$= \frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x| + C$$