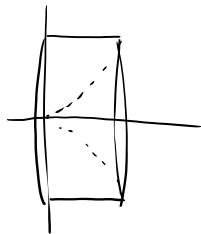
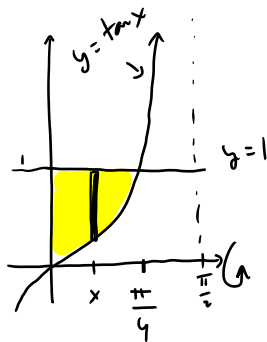


Section 6.2: Solid of Revolution, Disks and Washers

Example: The first quadrant region bounded by the line $y = 1$ and the curve $y = \tan x$ is rotated about the x -axis. Find the volume of the resulting solid.



outer radius $R_o = 1$

inner radius $R_i = \tan x$

$$V = \int_a^b \pi (R_o^2 - R_i^2) dx$$

Find b: set $y=1$ to $y=\tan x$

$$1 = \tan x \Rightarrow x = \frac{\pi}{4}$$

$$V = \int_0^{\pi/4} \pi (1^2 - \tan^2 x) dx$$

$$\tan^2 x = \sec^2 x - 1$$

$$= \pi \int_0^{\pi/4} (1 - (\sec^2 x - 1)) dx$$

$$= \pi \int_0^{\pi/4} (2 - \sec^2 x) dx$$

$$= \pi \left(2x - \tan x \right) \Big|_0^{\pi/4}$$

$$= \pi \left(2\left(\frac{\pi}{4}\right) - \tan\frac{\pi}{4} \right) - \pi \left(2 \cdot 0 - \tan 0 \right)$$

$$= \pi \left(\frac{\pi}{2} - 1 \right) = \frac{\pi^2}{2} - \pi$$

Suppose we wish to find the volume of the solid obtained by rotating the first quadrant region bounded by $y = x - x^3$ about the y -axis.

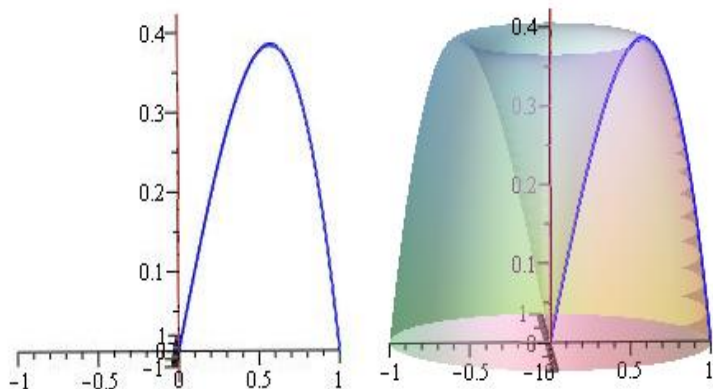


Figure: $y = x - x^3$, and the solid obtained from rotation about the y -axis.

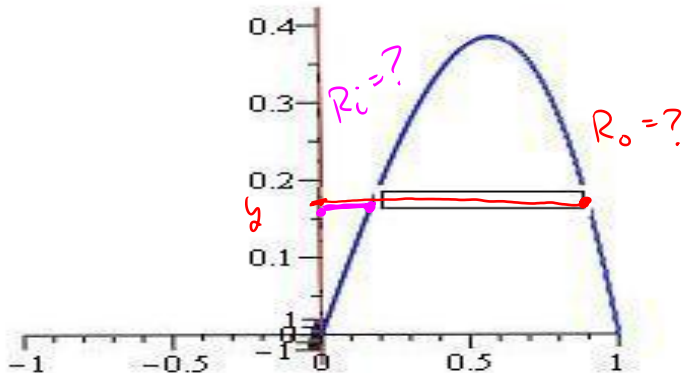


Figure: $y = x - x^3$. We can't find inner and outer radii (left and right functions) because we can't solve for x !!

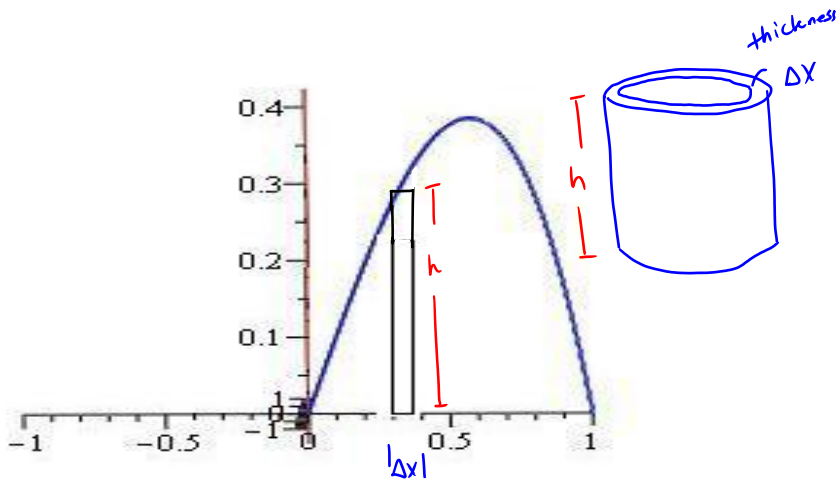


Figure: $y = x - x^3$, We can rotate vertical rectangles, but we won't get disks or washers!!

Section 6.3: Method of Cylindrical Shells

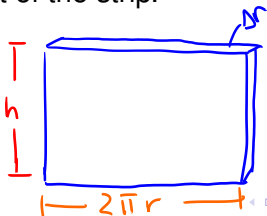
Suppose we take a thin strip (with width Δr) that is parallel to the axis of rotation. When we revolve this, we obtain a very thin cylindrical shell with volume

$$V = 2\pi rh\Delta r$$

where r is the average radius (distance between strip and the axis of rotation), and h is the height of the strip.



(1)



$$V = 2\pi rh\Delta r$$

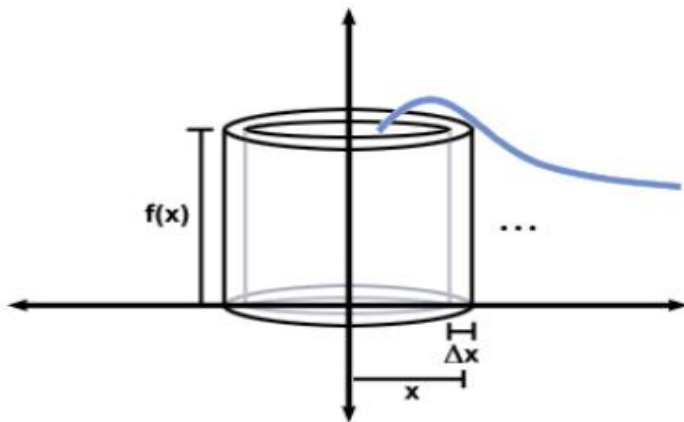


Figure: Revolving a vertical strip about a vertical axis creates a thin cylindrical shell.

Revolution about the x -axis: Method of Shells

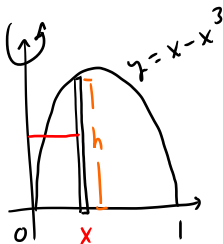
If the region is between the x -axis and the positive function $y = f(x)$, and the axis of revolution is the y -axis, then

$$\Delta r = \Delta x, \quad r = x, \quad \text{and} \quad h = f(x).$$

One shell has volume $V = 2\pi x f(x) \Delta x$. The whole solid has volume

$$V = \int_a^b 2\pi x f(x) dx$$

Find the volume of the solid obtained by rotating the first quadrant region bounded between $y = x - x^3$ and the x -axis about the y -axis.



$$V = \int_0^1 2\pi x f(x) dx$$

\uparrow \uparrow \uparrow
 r h ds

$$V = \int_0^1 2\pi x (x - x^3) dx$$

$$= 2\pi \int_0^1 (x^2 - x^4) dx$$

$$= 2\pi \left[\frac{x^3}{3} - \frac{x^5}{5} \right]_0^1$$

$$= 2\pi \left[\frac{1^3}{3} - \frac{1^5}{5} - \left(\frac{0^3}{3} - \frac{0^5}{5} \right) \right]$$

$$= 2\pi \left(\frac{1}{3} - \frac{1}{5} \right) = 2\pi \left(\frac{5-3}{15} \right) = \frac{4\pi}{15}$$

The region bound between $y = \sin(x^2)$ and the x -axis for $0 \leq x \leq \sqrt{\pi}$ is rotated about the y -axis. Find the volume of the resulting solid.

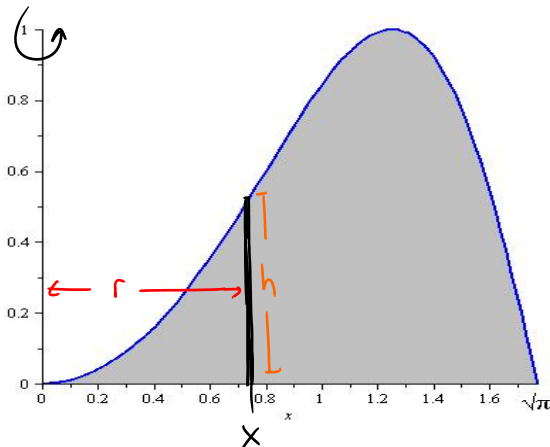


Figure: $y = \sin(x^2)$

$$V = \int_0^{\sqrt{\pi}} 2\pi x \sin(x^2) dx$$

$$= 2\pi \int_0^{\sqrt{\pi}} x \sin(x^2) dx$$

$$= \pi \int_0^{\pi} \sin(u) du$$

$$= \pi \left[-\cos u \right]_0^{\pi}$$

let
 $u = x^2$

$$du = 2x dx$$

when
 $x = 0, u = 0^2 = 0$

$$x = \sqrt{\pi}, u = (\sqrt{\pi})^2 = \pi$$

$$= \pi \left[-\cos \pi - (-\cos 0) \right]$$

$$= \pi \left[-(-1) + 1 \right] = 2\pi$$