March 9 MATH 1112 sec. 2 Spring 2020

Double & Half Angle IDs

Use $\sin(u+v) = \sin u \cos v + \sin v \cos u$ to obtain a formula for

$\sin(2u)$

Double Angle Formulas for the Cosine

Use $\cos(u+v) = \cos u \cos v - \sin u \sin v$ and $\cos^2 u + \sin^2 u = 1$ to find three formulas for

$$\begin{aligned}
\cos(2u) &= \cos u \cos u - \sin u \sin u \\
&= \cos^2 u - \sin^2 u \\
&= 2\cos^2 u - 1 \qquad (\text{because} \quad -\sin^2 u = \cos^2 u - 1) \\
&= 1 - 2\sin^2 u \qquad (\text{because} \quad \cos^2 u = 1 - \sin^2 u)
\end{aligned}$$

Question: Double Angle Formulas for the Tangent

From the sum formula $\tan(u+v) = \dfrac{\tan u + \tan v}{1 - \tan u \tan v}$, it follows that

$\tan(2u) =$

(a) $\dfrac{2\tan u}{1 - 2\tan u}$

(b) $\dfrac{2\tan u}{1 - \tan^2 u}$

(c) $\dfrac{\tan^2 u}{1 - 2\tan u}$

(d) $\dfrac{\tan^2 u}{1 - \tan^2 u}$

Double Angle Formulas

$$\begin{aligned}
\sin(2u) &= 2\sin u \cos u \\
\cos(2u) &= \cos^2 u - \sin^2 u \\
&= 2\cos^2 u - 1 \\
&= 1 - 2\sin^2 u \\
\tan(2u) &= \frac{2\tan u}{1 - \tan^2 u}
\end{aligned}$$

Example

Suppose $\csc(x) = 5$ and $\cot(x) < 0$. Find the exact value of

$\sec(2x)$

Half Angle IDs

Use $\cos(2x) = 1 - 2\sin^2 x$ to find $\sin^2\left(\frac{\pi}{8}\right)$.

Half Angle IDs

$$\sin^2 x = \frac{1-\cos(2x)}{2} \qquad \sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos x}{2}}$$

$$\cos^2 x = \frac{1+\cos(2x)}{2} \qquad \cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+\cos x}{2}}$$

$$\tan^2 x = \frac{1-\cos(2x)}{1+\cos(2x)} \qquad \tan\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos(x)}{1+\cos(x)}}$$

For a given value of x, only one of the signs $+$ or $-$ will apply. To choose the correct sign, determine which quadrant the angle $\frac{x}{2}$ is in when in standard position.

Determine the exact value of

(a) $\cos(22.5^\circ)$

Question

Suppose θ is an acute angle and $\cos\theta = \frac{1}{3}$. Then $\cos\left(\frac{\theta}{2}\right)$ is equal to

(a) $\frac{1}{6}$

(b) $\pm\sqrt{\frac{2}{3}}$

(c) $\sqrt{\frac{1}{3}}$

(d) $\sqrt{\frac{2}{3}}$

(e) can't be determined without more information