# March 3 Math 2306 sec 59 Spring 2016

#### Section 9: Method of Undetermined Coefficients

The context here is linear, constant coefficient, nonhomogeneous equations

$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_0 y = g(x)$$

where g comes from the restricted classes of functions

- polynomials,
- exponentials,
- sines and/or cosines,
- and products and sums of the above kinds of functions

Recall  $y = y_c + y_p$ , so we'll have to find both the complementary and the particular solutions!

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## Some Motivating Examples

Find a particular solution of the ODE

$$y'' - 4y' + 4y = 8x + 1$$

We made a guess that  $y_p = Ax + B$ , a line, since the right hand side is a line. We found that this worked with A = 2 and B = 9/4 giving the particular solution  $y_p = 2x + 9/4$ .

$$y'' - 4y' + 4y = 6e^{3x}$$

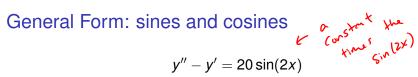
We made a guess that  $y_p = Ae^{3x}$  which matches the basic form of the right hand side. We found that this worked with A = 6 giving the particular solution  $y_p = 6e^{3x}$ .

### Make the form general

$$y'' - 4y' + 4y = 16x^2$$

With this one, we tried setting  $y_p = Ax^2$ , but we couldn't solve the equation for any value of *A*.

We corrected it to the form  $y_p = Ax^2 + Bx + C$  and found that this does solve the equation if A = 4, B = 8 and C = 6. So we found the particular solution  $y_p = 4x^2 + 8x + 6$ .



$$y''-y'=20\sin(2x)$$

Let's suppose 
$$y_p = A \sin(2x)$$
.  
 $y_p = A \sin(2x)$   
 $y_p' = 2A \cos(2x)$   
 $y_p'' = -y_p' = 20 \sin(2x)$   
 $y_p'' = -y_p' = 20 \sin(2x)$   
 $-YA \sin(2x) - 2A \cos(2x) = 20 \sin(2x)$   
 $Matching (oeff:creats, use nud)$   
 $-YA = 20 ond -2A = 0$ 

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We may need to match coefficients for Cos(2x). Yp= ASin(2x) + BCos(2x) 5. Let's guess yp'= 2A (os (2x) - 2BS in (2x) yp" = - 4ASin(2x) - 4B (w (2x) yp" - yp = ZO Sin (2x) we require -4ASin(2x)-4BCos(2x)-(2ACos(2x)-2BSin(2x))=2OSin(2x)(-4A + 2B) Sin (2x) + (-2A - 4B) Cus (2x) = 20 Sin $(2x) + 0 \cdot Cus(2x)$ 

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$$-4A + 2B = 20 \implies -8A + 4B = 40$$

$$-2A - 4B = 0 \qquad add$$

$$-10A = 40 \implies A = -4$$

$$-4B-2A = 2(-4)=-8 \implies B-2$$

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# Examples of Forms of $y_p$ based on g (Trial Guesses)

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(a) g(x) = 1 (or really any nonzero constant)

Guess yp= A

(b) 
$$g(x) = x - 7$$
 ist degree Poly.  $y_p = A \times + B$ 

(c) 
$$g(x) = 5x$$
 <sup>152</sup> degree Polo. Sp =  $A \times + B$ 

(d) 
$$g(x) = 3x^3 - 5$$
  $3^{rd}$  degree poly  
 $y_p = Ax^3 + Bx^2 + Cx + D$ 

More Trial Guesses  
(e) 
$$g(x) = xe^{3x}$$
  
 $y_p = (Ax+B)e^{3x} = Axe^{3x} + Be^{3x}$   
(f)  $g(x) = \cos(7x)$  Sum of sines / cosines of 7x  
 $y_p = A \cos(7x) + B \sin(7x)$   
(g)  $g(x) = \sin(2x) - \cos(4x)$   
 $y_p = A \cos(7x) + B \sin(7x)$   
(g)  $g(x) = \sin(2x) - \cos(4x)$   
 $y_p = A \cos(7x) + B \sin(7x)$   
(h)  $g(x) = x^2 \sin(3x)$   
 $z^{nb} degue poly times sines / cosines 3x$   
 $y_p = (Ax + Bx + C) \sin(3x) + (Dx^2 + Ex + F) \cos(3x)$ 

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#### Still More Trial Guesses

(i) 
$$g(x) = e^x \cos(2x)$$
  $y_p = Ae^x \cos(2x) + Be^x \sin(2x)$ 

(j) 
$$g(x) = x^3 e^{8x}$$
  $y_{p^2} \left( A_{X^3}^3 B_{X^4}^2 (x+1) \right) e^{8x}$ 

(k) 
$$g(x) = xe^{-x}\sin(\pi x)$$
  
 $y_{\rho^{z}}(Ax+B)e^{-x}Sin(\pi x) + (Cx+D)e^{-x}Cus(\pi x)$ 

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#### The Superposition Principle

$$y'' - y' = 20\sin(2x) + 4e^{-5x}$$

We can consider  $y'' - y' = g_1(x) + g_2(x)$ where  $g_1(x) = 20 \sin(2x)$  and  $g_2(x) = 4e^{5x}$ 

For  $y'' - y' = 4e^{5x}$ , guess  $y_{p_{2}}^{z} = Ae^{5x}$  $y_{p_{2}}^{z} = -SAe^{-5x}$ ,  $y_{p_{2}}^{z}'' = 2^{5}Ae^{-5x}$ 

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