March 4 MATH 1112 sec. 52 Spring 2020

Inverse Trigonometric Functions

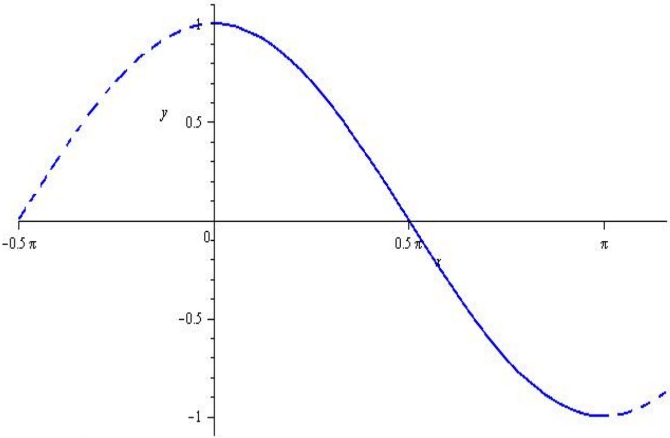


Figure: **Inverse Cosine:** To define an inverse cosine function, we start by restricting the domain of $\cos(x)$ to the interval $[0, \pi]$

The Inverse Cosine Function (a.k.a. arccosine function)

Definition: For x in the interval $[-1, 1]$ the inverse cosine of x is denoted by either

$$\cos^{-1}(x) \quad \text{or} \quad \arccos(x)$$

and is defined by the relationship

$$y = \cos^{-1}(x) \quad \Longleftrightarrow \quad x = \cos(y) \quad \text{where} \quad 0 \le y \le \pi.$$

The Domain of the Inverse Cosine is $-1 \le x \le 1$.

The Range of the Inverse Cosine is $0 \le y \le \pi$.

Quad I and II

The Graph of the Arccosine

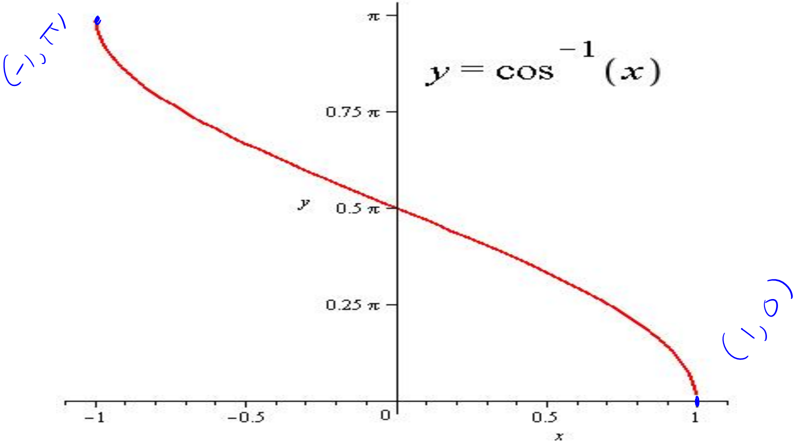


Figure: Note that the domain is $-1 \leq x \leq 1$ and the range is $0 \leq y \leq \pi$.

Function/Inverse Function Relationship

For every x in the interval $[-1, 1]$

$$\cos\left(\cos^{-1}(x)\right) = x$$

For every x in the interval $[0, \pi]$

$$\cos^{-1}\left(\cos(x)\right) = x$$

Remark 1: If $x > 1$ or $x < -1$, the expression $\cos^{-1}(x)$ is not defined.

Remark 2: If $x > \pi$ or $x < 0$, the expression $\cos^{-1}\left(\cos(x)\right)$ IS defined, but IS NOT equal to x.

Some Inverse Cosine Values

We can build a table of some inverse cosine values by using our knowledge of the cosine function.

x	$\cos(x)$
0	1
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{2}$	0
$\frac{2\pi}{3}$	$-\frac{1}{2}$
$\frac{5\pi}{6}$	$-\frac{\sqrt{3}}{2}$
π	-1

x	$\cos^{-1}(x)$
1	0
$\frac{\sqrt{3}}{2}$	$\frac{\pi}{6}$
$\frac{1}{\sqrt{2}}$	$\frac{\pi}{4}$
0	$\frac{\pi}{2}$
$-\frac{1}{2}$	$\frac{2\pi}{3}$
$-\frac{\sqrt{3}}{2}$	$\frac{5\pi}{6}$
-1	π

Conceptual Definition

We can think of the inverse cosine function in the following way:

$\cos^{-1}(x)$ **is the *angle* between** 0 **and** π **whose cosine is** x**.**

quadrants I or II

Examples

Evaluate each expression exactly.

(a) $\cos^{-1}(0) = \frac{\pi}{2}$

angle θ in $[0,\pi]$ such that $\cos\theta = 0$

(b) $\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

we need θ in $[0,\pi]$ with $\cos\theta = \frac{-1}{2}$

$\theta = \frac{2\pi}{3}$

Question

The exact value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) =$

(a) $\frac{\pi}{4}$

(b) $-\frac{\pi}{4}$

(c) $\frac{3\pi}{4}$

(d) $-\frac{3\pi}{4}$

Examples

Evaluate each expression if possible. If undefined, state a reason.

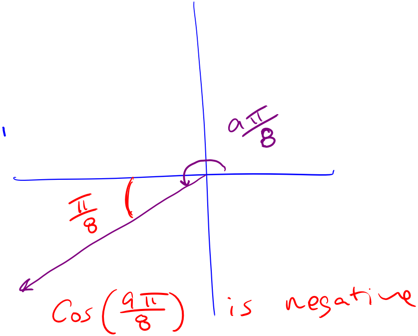
(a) $\cos\left[\cos^{-1}\left(\frac{1}{4}\right)\right]$ $= \frac{1}{4}$

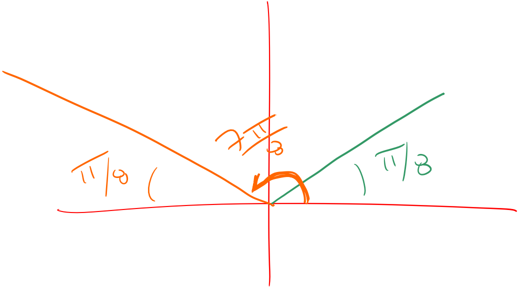
(b) $\cos\left[\cos^{-1}(6)\right]$ - undefined

6 is not in the domain of $\cos^{-1} x$

Evaluate each expression if possible. If undefined, state a reason.

(c) $\cos^{-1}\left[\cos\left(\frac{9\pi}{8}\right)\right] = \frac{7\pi}{8}$





Question

Evaluate $\cos^{-1}\left[\cos\left(-\frac{\pi}{4}\right)\right]$

(a) $\frac{\pi}{4}$

(b) $-\frac{\pi}{4}$

(c) $\frac{3\pi}{4}$

(d) $-\frac{3\pi}{4}$

The Inverse Tangent Function

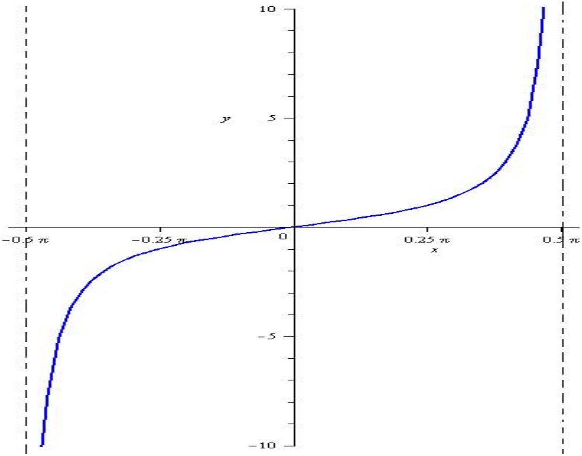


Figure: To define an inverse tangent function, we start by restricting the domain of $\tan(x)$ to the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. **(Note the end points are NOT included!)**

The Inverse Tangent Function (a.k.a. arctangent function)

Definition: For all real numbers x, the inverse tangent of x is denoted by

$$\tan^{-1}(x) \quad \text{or by} \quad \arctan(x)$$

and is defined by the relationship

$$y = \tan^{-1}(x) \quad \Longleftrightarrow \quad x = \tan(y) \quad \text{where} \quad -\frac{\pi}{2} < y < \frac{\pi}{2}.$$

The Domain of the Inverse Tangent is $-\infty < x < \infty$**.**

The Range of the Inverse ~~Cosine~~ is $-\frac{\pi}{2} < y < \frac{\pi}{2}$ **(Note the strict inequalities.).**

The Graph of the Arctangent

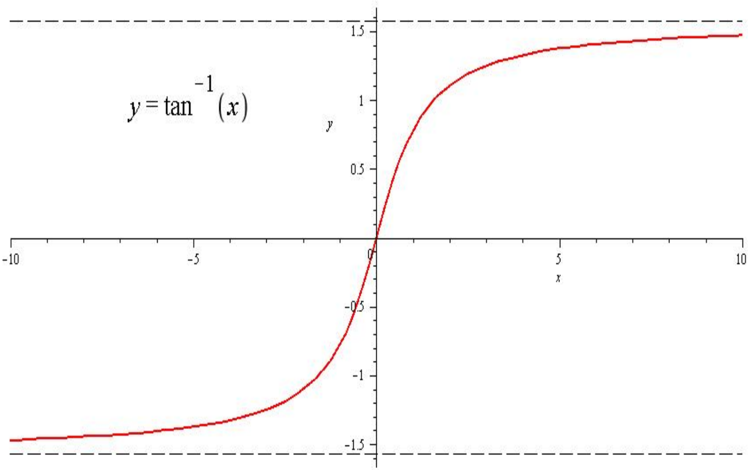


Figure: The domain is all real numbers and the range is $-\frac{\pi}{2} < y < \frac{\pi}{2}$. The graph has two horizontal asymptotes $y = -\frac{\pi}{2}$ and $y = \frac{\pi}{2}$.

Function/Inverse Function Relationship

For all real numbers x

$$\tan\left(\tan^{-1}(x)\right) = x$$

For every x in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$$\tan^{-1}\left(\tan(x)\right) = x$$

Remark 1: The expression $\tan^{-1}(x)$ is always well defined.

Remark 2: If $x > \frac{\pi}{2}$ or $x < -\frac{\pi}{2}$, the expression $\tan^{-1}\left(\tan(x)\right)$ MAY BE defined, but IS NOT equal to x.

Some Inverse Tangent Values

We can build a table of some inverse tangent values by using our knowledge of the tangent function.

x	$\tan(x)$
$-\frac{\pi}{3}$	$-\sqrt{3}$
$-\frac{\pi}{4}$	-1
$-\frac{\pi}{6}$	$-\frac{1}{\sqrt{3}}$
0	0
$\frac{\pi}{6}$	$\frac{1}{\sqrt{3}}$
$\frac{\pi}{4}$	1
$\frac{\pi}{3}$	$\sqrt{3}$

x	$\tan^{-1}(x)$
$-\sqrt{3}$	$-\frac{\pi}{3}$
-1	$-\frac{\pi}{4}$
$-\frac{1}{\sqrt{3}}$	$-\frac{\pi}{6}$
0	0
$\frac{1}{\sqrt{3}}$	$\frac{\pi}{6}$
1	$\frac{\pi}{4}$
$\sqrt{3}$	$\frac{\pi}{3}$

Conceptual Definition

We can think of the inverse tangent function in the following way:

$\tan^{-1}(x)$ is the *angle* between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ whose tangent is x.

Quad I and IV