March 15 MATH 1112 sec. 54 Spring 2019

Let's Recall the basic plots $y = \sin x$ and $y = \cos x$

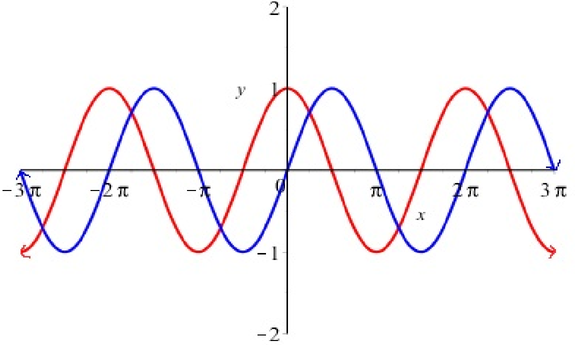


Figure: Some points on the sine graph are $(0, 0)$, $(\frac{\pi}{2}, 1)$, $(\pi, 0)$, $(\frac{3\pi}{2}, -1)$, $(2\pi, 0)$.
Some points on the cosine graph are $(0, 1)$, $(\frac{\pi}{2}, 0)$, $(\pi, -1)$, $(\frac{3\pi}{2}, 0)$, $(2\pi, 1)$

Section 6.6: Graphing Trigonometric Functions with Transformations

Our goal is to graph functions of the form

$$f(x) = a\sin(bx - c) + d \quad \text{or} \quad f(x) = a\cos(bx - c) + d$$

Note: here we will be graphing points (x, y) on a curve $y = f(x)$.

Amplitude

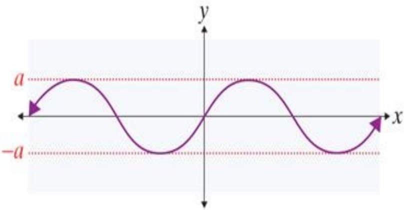
Consider: $f(x) = a\sin(bx - c) + d$ or $f(x) = a\cos(bx - c) + d$

Definition: Let a be any nonzero real number. The **amplitude** of the function f defined above is the value $|a|$.

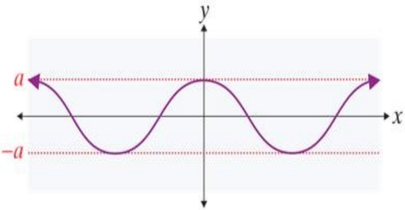
Recall that this is half the distance between the maximum and minimum values.

If $a < 0$ the graph is reflected in the x-axis. But the amplitude is still $|a|$.

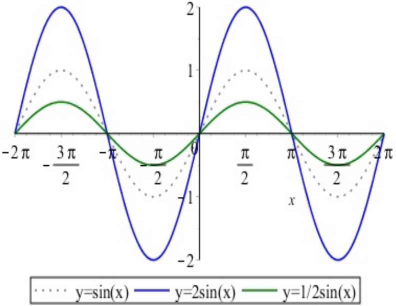
Amplitude

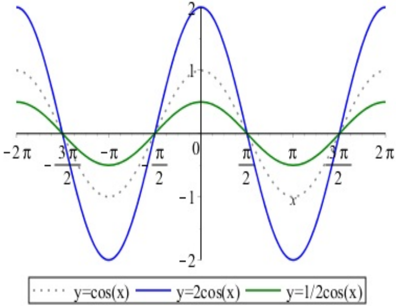


$f(x) = a\sin x$



$f(x) = a\cos x$





Example

Identify the amplitude A of each function. Determine if the graph is reflected in the x-axis.

(a) $f(x) = 3\sin(4x-2)+1$ $A = |3| = 3$

(b) $f(x) = 2-6\cos(2x+3)$ $A = |-6| = 6$

Question

The amplitude A of the function $y = 2 - \sin(3x + 1)$ is

(a) 1

$A = |-1| = 1$

(b) -1

(c) 3

(d) 2

Period

Consider: $f(x) = a\sin(bx - c) + d$ or $f(x) = a\cos(bx - c) + d$

Theorem: Let b be any positive real number. The **fundamental period** of the function f above is given by

$$T = \frac{2\pi}{|b|}.$$

Recall that the fundamental period of $\cos x$ and $\sin x$ is 2π.

Period

Consider: $f(x) = a\sin(bx - c) + d$ or $f(x) = a\cos(bx - c) + d$

Due to symmetry, we can always assume $b > 0$. Note for example

$$\sin(-3x + 2) = \sin(-(3x - 2)) = -\sin(3x - 2).$$

The period is **always positive.** The period in this example is $\frac{2\pi}{3}$. Based on this, some authors say that $b > 0$ and that the period

$$T = \frac{2\pi}{b}$$

Period

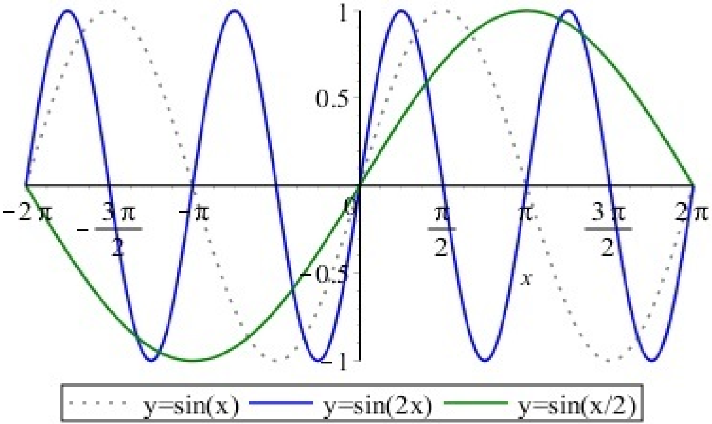


Figure: Comparisons with $b = 1/2, 1$, and 2. On the interval $-2\pi < x < 2\pi$ we obtain one ($b = 1/2$), two ($b = 1$) or four ($b = 2$) full cycles.

Example

Identify the period of each function. Call the period T

(a) $f(x) = 3\sin(4x-2)+1$

$b = 4$ so $T = \frac{2\pi}{4} = \frac{\pi}{2}$

(b) $f(x) = -5\sin\left(\frac{\pi x}{2}\right)+7$

$b = \frac{\pi}{2}$ so $T = \frac{2\pi}{\pi/2} = 2\pi\left(\frac{2}{\pi}\right) = 4$

Frequency

Consider: $f(x) = a\sin(bx - c) + d$ or $f(x) = a\cos(bx - c) + d$

Definition: The reciprocal of the period is called the **frequency**. That is

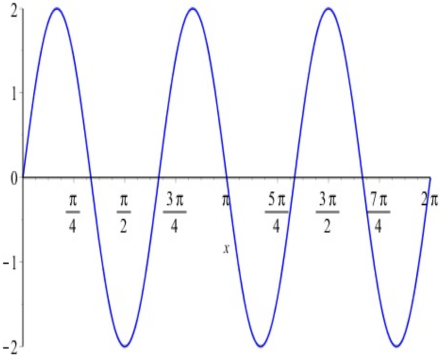
$$\text{frequency} = \frac{1}{T} = \frac{b}{2\pi}.$$

If x represents time, then

- the period tells us how much time is required for one full cycle, and
- the frequency tells us how many cycles occur in one time unit.

If $y = \cos(bx)$ (or $y = \sin(bx)$), then b the number of cycles occuring in an interval of length 2π.

Question



The figure shows $y = f(x)$ for $0 < x < 2\pi$. Which of the following is true?

(a) $f(x) = 2\sin\left(\frac{1}{3}x\right)$

(b) $f(x) = 2\sin(3x)$

(c) $f(x) = \frac{1}{2}\sin\left(\frac{1}{3}x\right)$

(d) $f(x) = \frac{1}{2}\sin(3x)$

Figure: Hint: Count the number of full cycles.

Phase Shift (horizontal shift)

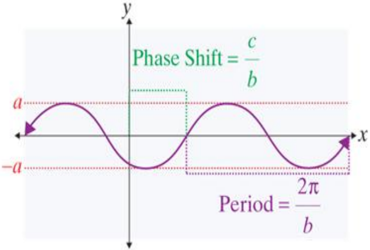
Consider: $f(x) = a\sin(bx - c) + d$ or $f(x) = a\cos(bx - c) + d$

Definition: A horizontal shift is called a **phase shift.** Again assuming that $b > 0$, the phase shift for f above is

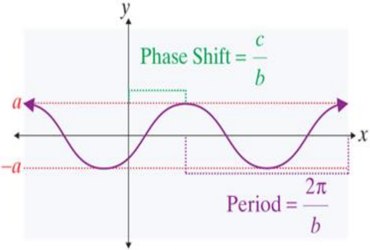
$$\frac{|c|}{b} \text{ units}$$

$\sin\left(b\left(x - \frac{c}{b}\right)\right)$

to the right if $c > 0$ and to the left if $c < 0$.



$f(x) = a\sin(bx - c)$



$f(x) = a\cos(bx - c)$

Question

Recall that for $f(x) = a\sin(bx - c) + d$, the phase shift is $\left|\frac{c}{b}\right|$

The phase shift of $y = 3\cos\left(2x - \frac{\pi}{2}\right)$ is

(a) π and to the right

(b) $\frac{\pi}{3}$ and to the right

(c) $\frac{\pi}{4}$ and to the right

(d) $\frac{\pi}{4}$ and to the left

(e) π and to the left

$c = \frac{\pi}{2}$, $b = 2$

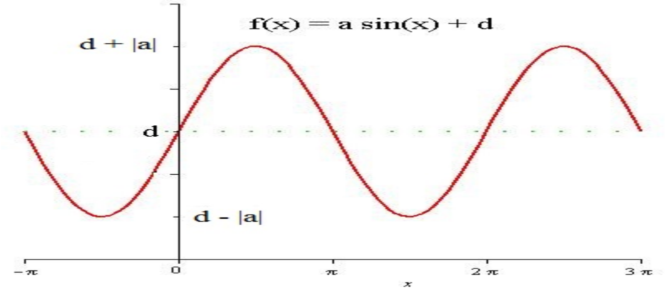
$\frac{c}{b} = \frac{\pi/2}{2} = \frac{\pi}{4}$

$c > 0$

Vertical Shift

Consider: $f(x) = a\sin(bx - c) + d$ or $f(x) = a\cos(bx - c) + d$

Definition: If d is a nonzero number, then the function f has a **vertical shift** of $|d|$ units up if $d > 0$ and down if $d < 0$.



Question

Regarding the function $f(x) = -5\sin\left(\frac{\pi x}{2}\right) - 7$, there is a

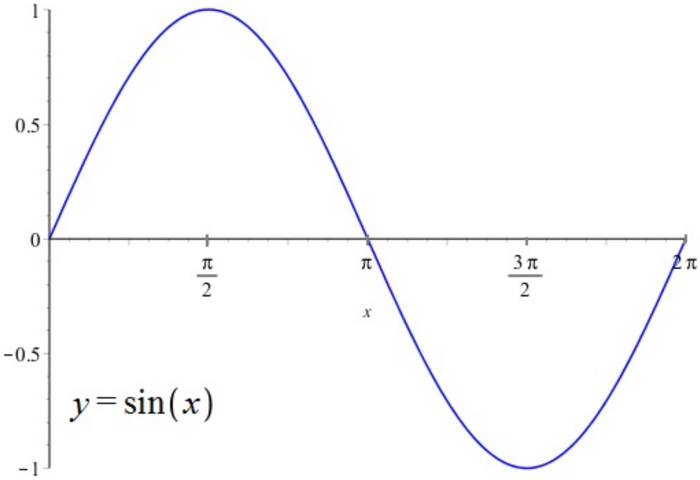
(a) vertical shift down 5 units

(b) vertical shift down 7 units

(c) vertical shift up 7 units

(d) vertical shift down 12 units

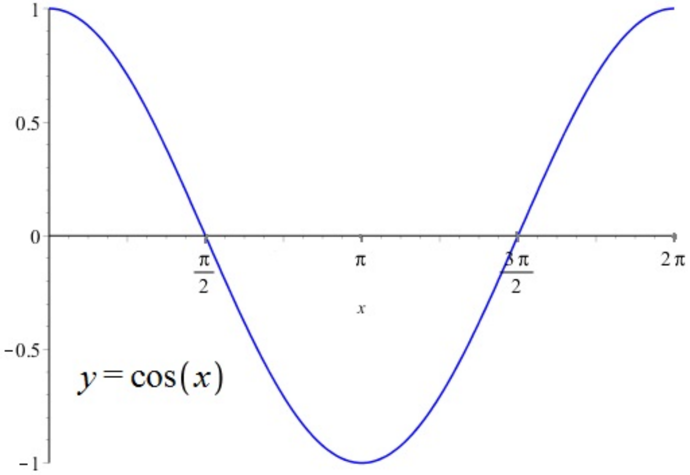
Parent Plots



The period can be divided into four equal segments.

For the sine function x-int $\rightarrow$ max $\rightarrow$ x-int $\rightarrow$ min $\rightarrow$ x-int

Parent Plots



The period can be divided into four equal segments.

For the cosine function max → x-int → min → x-int → max

Pulling it all Together!

Plot two full periods of the function $f(x) = a\sin(bx - c) + d$ (or $f(x) = a\cos(bx - c) + d$). Carry out each of the following steps:

- Identify the amplitude and determine if there is an x-axis reflection.
- Identify the period. Find the length of one fourth of the period.
- Identify any phase shift with its direction. Identify end points and points that divide the period into four equal parts.
- Identify any vertical shift with its direction.
- Use the basic plot of $y = \sin x$ or $y = \cos x$ to get the profile.

$f(x) = 2 - 4\cos\left(\pi x - \frac{\pi}{2}\right)$ $= -4\cos\left(\pi x - \frac{\pi}{2}\right) + 2$

Identify the amplitude and period. Determine if there is a reflection in the x-axis. Find the length of one quarter period.

(not really → x-axis)

$a = -4$, $b = \pi$, $c = \pi/2$, $d = 2$

Amplitude $A = |-4| = 4$

$-4 < 0$, so there is a horizontal reflection

Period $T = \frac{2\pi}{b} = \frac{2\pi}{\pi} = 2$

A quarter period is $\frac{2}{4} = \frac{1}{2}$ units

$f(x) = 2 - 4\cos\left(\pi x - \frac{\pi}{2}\right)$

Identify any phase shift and vertical shift along with direction.
Determine the end points and points that divide the period into four parts.

$c = \frac{\pi}{2}$ and $b = \pi$

So there is a phase shift $\left|\frac{c}{b}\right| = \frac{\pi/2}{\pi} = \frac{1}{2}$

to the right

$d = 2$ so there is a vertical shift up 2.

End points for a period are

left @ $x = \frac{1}{2}$, right @ $x = \frac{1}{2} + 2 = \frac{5}{2}$

$f(x) = 2 - 4\cos\left(\pi x - \frac{\pi}{2}\right)$

Plot two periods of its graph.