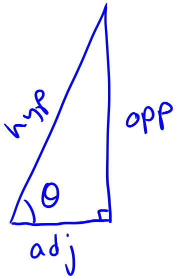
March 1 MATH 1112 sec. 54 Spring 2019

Sections 6.1 & 6.2: Trigonometric Functions of Acute Angles

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}, \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}}$$

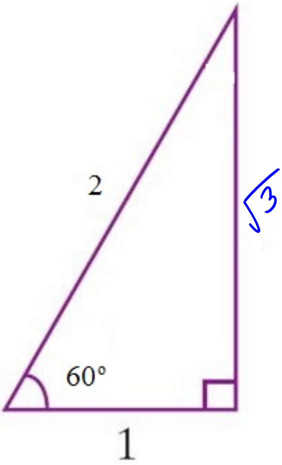
$$\cos\theta = \frac{\text{adj}}{\text{hyp}}, \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}}, \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$$



Question

Half of an equilateral triangle is a right triangle with angles $30^\circ, 60^\circ$ and 90°. The ratio of the base to the hypotenuse is $1:2$. Based on this, which of the following is true?



(a) $\sin 60^\circ = \frac{1}{2}$ and $\cos 60^\circ = 2$

(b) $\sin 60^\circ = \frac{1}{2}$ and $\cos 60^\circ = \frac{\sqrt{3}}{2}$

(c) $\sin 60^\circ = \frac{1}{2}$ and $\cos 60^\circ = \frac{1}{2}$

(d) $\sin 60^\circ = \frac{\sqrt{3}}{2}$ and $\cos 60^\circ = \frac{1}{2}$

Some Key Trigonometric Values

Use the triangles to determine the six trigonometric values of the angles $30^\circ, 45^\circ$, and 60°. Those are $\frac{\pi}{6}, \frac{\pi}{4}$, and $\frac{\pi}{3}$, respectively.

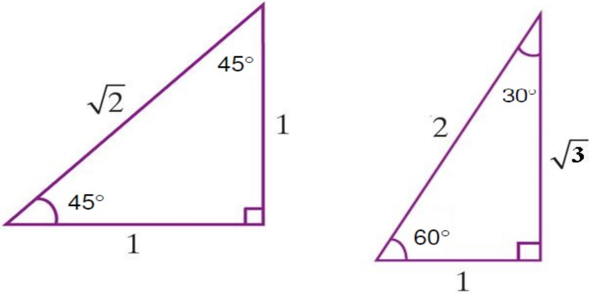


Figure: An isosceles right triangle of leg length 1 (left), and half of an equilateral triangle of side length 2 (right).

Commit To Memory

It is to our advantage to remember the following:

$$\sin 30^\circ = \frac{1}{2}, \qquad \sin 45^\circ = \frac{1}{\sqrt{2}}, \qquad \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2}, \qquad \cos 45^\circ = \frac{1}{\sqrt{2}}, \qquad \cos 60^\circ = \frac{1}{2}$$

$$\tan 30^\circ = \frac{1}{\sqrt{3}}, \qquad \tan 45^\circ = 1, \qquad \tan 60^\circ = \sqrt{3}$$

We'll use these to find some other trigonometric values. Still others will require a calculator.

Commit To Memory

These are the same trigonometric values stated in radians:

$$\sin\frac{\pi}{6}=\frac{1}{2},\qquad \sin\frac{\pi}{4}=\frac{1}{\sqrt{2}},\qquad \sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$$

$$\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2},\qquad \cos\frac{\pi}{4}=\frac{1}{\sqrt{2}},\qquad \cos\frac{\pi}{3}=\frac{1}{2}$$

$$\tan\frac{\pi}{6}=\frac{1}{\sqrt{3}},\qquad \tan\frac{\pi}{4}=1,\qquad \tan\frac{\pi}{3}=\sqrt{3}$$

We'll use these to find some other trigonometric values. Still others will require a calculator.

Calculator

Figure: Any scientific calculator will have built in functions for sine, cosine and tangent. (TI-84 shown)

Using a Calculator

Evaluate the following using a calculator. Round answers to three decimal places.

$\sin 16^\circ =$ 0.276

$\sec 78.3^\circ =$ $\dfrac{1}{\cos(78.3^\circ)} = 4.931$

$\tan\left(\dfrac{2\pi}{7}\right) =$ 1.254

Question

Use a calculator to evaluate $\sec(57^\circ)$ to four decimal places.

(a) 1.1113

(b) 0.5446

(c) 1.8361

(d) This can't be done since the calculator doesn't have a secant button.

Angle of Elevation, Angle of Depression

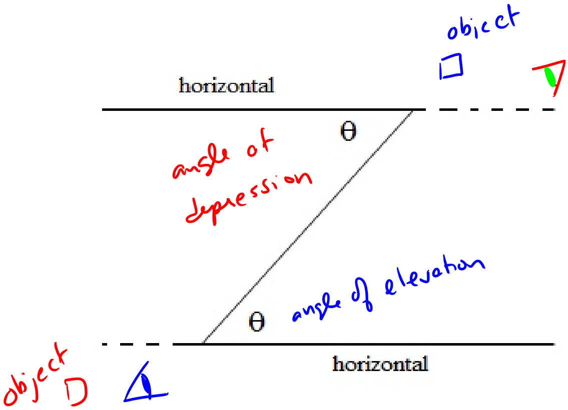
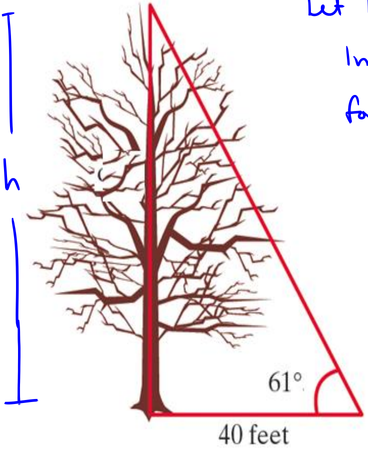


Figure: An observer may view an object at an angle of elevation or depression with respect to a horizontal vantage point.

Application Example

Before cutting down a dead tree, you wish to determine its height. From a horizontal distance of 40 ft, you measure the angle of elevation from the ground to the top of the tree to be 61°. Determine the tree height to the nearest 100^{th} of a foot.



Let h be the tree height.

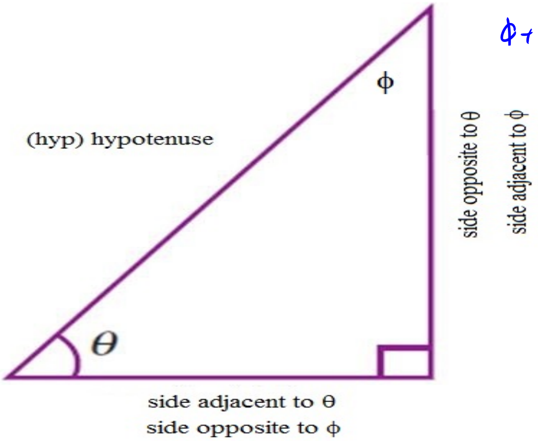
In the diagram h = opp and 40 ft = adj for the 61° angle

$$\tan 61^\circ = \frac{\text{opp}}{\text{adj}} = \frac{h}{40\text{ft}}$$

$$h = 40\text{ft}(\tan 61^\circ) \approx 72.16 \text{ ft}$$

Complementary Angles and Cofunction Identities

The two acute angles in a right triangle are complementary angles.



$\phi + \theta = 90^\circ$

$\theta = 90^\circ - \phi$

$\phi = 90^\circ - \theta$

Figure: Note that for complementary angles θ and ϕ, the role of the legs (opposite versus adjacent) are interchanged.

Cofunction Identities

For any acute angle θ

$$\sin\theta = \cos(90^\circ - \theta) \qquad \cos\theta = \sin(90^\circ - \theta)$$

$$\tan\theta = \cot(90^\circ - \theta) \qquad \cot\theta = \tan(90^\circ - \theta)$$

$$\sec\theta = \csc(90^\circ - \theta) \qquad \csc\theta = \sec(90^\circ - \theta)$$

Stated in radians

$$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right) \qquad \cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$$

$$\tan\theta = \cot\left(\frac{\pi}{2} - \theta\right) \qquad \cot\theta = \tan\left(\frac{\pi}{2} - \theta\right)$$

$$\sec\theta = \csc\left(\frac{\pi}{2} - \theta\right) \qquad \csc\theta = \sec\left(\frac{\pi}{2} - \theta\right)$$

These equations define what are called **cofunction identities.**

Question

Suppose θ is an acute angle such that $\sin\theta = 0.334$. Which of the following is true?

$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$

(a) $\sin\left(\frac{\pi}{2} - \theta\right) = 1 - 0.334$

(b) $\cos\left(\frac{\pi}{2} - \theta\right) = 0.334$

(c) $\csc\left(\frac{\pi}{2} - \theta\right) = 0.334$

(d) $\csc\theta = \dfrac{1}{1 - 0.334}$

(e) There's not enough information to determine whether any of the above is true.