March 29 MATH 1112 sec. 54 Spring 2019

Section 7.1: Fundamental Identities: Pythagorean, Sum, and Difference

We have three new identities added to our list. The Pythagorean IDs are

$$\sin^2 x + \cos^2 x = 1$$

$$\tan^2 x + 1 = \sec^2 x$$

and

$$1 + \cot^2 x = \csc^2 x$$

We also added the sum and difference of angles identities.

Sum and Difference Identities

Cosine Identities:

$$\text{(sum)} \quad \cos(u+v) = \cos u \cos v - \sin u \sin v,$$
$$\text{(diff)} \quad \cos(u-v) = \cos u \cos v + \sin u \sin v$$

Sine Identities:

$$\text{(sum)} \quad \sin(u+v) = \sin u \cos v + \sin v \cos u$$
$$\text{(diff)} \quad \sin(u-v) = \sin u \cos v - \sin v \cos u$$

Tangent Identities:

$$\text{(sum)} \quad \tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v},$$
$$\text{(diff)} \quad \tan(u-v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

Section 7.2: Double & Half Angle IDs

Next we derived the double angle identities:

$$\sin(2u) = 2\sin u\cos u$$

$$\begin{aligned}\cos(2u) &= \cos^2 u - \sin^2 u\\ &= 2\cos^2 u - 1\\ &= 1 - 2\sin^2 u\end{aligned}$$

$$\tan(2u) = \frac{2\tan u}{1-\tan^2 u}$$

Example

Suppose $\sec(x) = 5$ and $\cot(x) < 0$. Find the exact value of

$\csc(2x)$ $= \dfrac{1}{\sin(2x)}$ $\sin(2x) = 2\sin x \cos x$

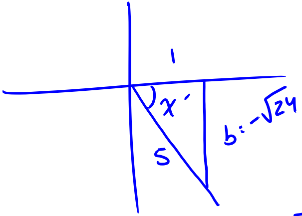
Note $\sec x = \dfrac{1}{\cos x}$ so $\cos x = \dfrac{1}{\sec x} = \dfrac{1}{5}$

we still need $\sin x$.

$\sec x > 0$ and $\cot x < 0$

quad I or $\underline{\overline{\text{IV}}}$ quad II or $\underline{\overline{\text{IV}}}$

x is in quadrant IV in standard pos.



$$\sec x = 5 = \frac{hyp}{adj}$$

$$b^2 = 5^2 - 1^2 = 24$$

From the diagram $\sin x = \frac{-\sqrt{24}}{5}$

$$\csc(2x) = \frac{1}{\sin(2x)} = \frac{1}{2\sin x \cos x} = \frac{1}{2\left(\frac{-\sqrt{24}}{5}\right)\left(\frac{1}{5}\right)}$$

$$= \frac{1}{\frac{-2\sqrt{24}}{25}} = \frac{-25}{2\sqrt{24}}$$

Half Angle IDs

Use the fact that $\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ and that $\frac{\pi}{4} = 2\frac{\pi}{8}$ to find the exact value of $\sin^2\left(\frac{\pi}{8}\right)$

We'll relate $\cos\left(2\cdot\frac{\pi}{8}\right) = \cos\left(\frac{\pi}{4}\right)$ to the $\sin\left(\frac{\pi}{8}\right)$. One formula for $\cos(2u)$ is

$$\cos(2u) = 1 - 2\sin^2 u$$

We'll take $u = \frac{\pi}{8}$. Isolate $\sin^2 u$

$$2\sin^2 u = 1 - \cos(2u)$$

$$\sin^2 u = \frac{1-\cos(2u)}{2}$$

Set $u = \frac{\pi}{8}$

$$\sin^2\left(\frac{\pi}{8}\right) = \frac{1-\cos\left(\frac{\pi}{4}\right)}{2}$$

$$= \frac{1-\frac{1}{\sqrt{2}}}{2} \cdot \left(\frac{\sqrt{2}}{\sqrt{2}}\right)$$

$$= \frac{\sqrt{2}-1}{2\sqrt{2}}$$

Half Angle IDs

$$\sin^2 x = \frac{1-\cos(2x)}{2} \qquad \sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos x}{2}}$$

$$\cos^2 x = \frac{1+\cos(2x)}{2} \qquad \cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+\cos x}{2}}$$

$$\tan^2 x = \frac{1-\cos(2x)}{1+\cos(2x)} \qquad \tan\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos(x)}{1+\cos(x)}}$$

For a given value of x, only one of the signs $+$ or $-$ will apply. To choose the correct sign, determine which quadrant the angle $\frac{x}{2}$ is in when in standard position.

Determine the exact value of

(a) $\cos(22.5^\circ)$

$22.5^\circ = \frac{45^\circ}{2}$

we'll use $\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+\cos x}{2}}$

22.5° is acute so $\cos(22.5^\circ) > 0$ (plus sign)

$$\cos(22.5^\circ) = \sqrt{\frac{1+\cos(45^\circ)}{2}} = \sqrt{\frac{1+\frac{1}{\sqrt{2}}}{2}}$$

$$= \sqrt{\left(\frac{1+\frac{1}{\sqrt{2}}}{2}\right)\frac{\sqrt{2}}{\sqrt{2}}} = \sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}$$

Question

In standard position, the angle $\frac{13\pi}{12}$ would have its terminal side in

(a) Quadrant 1

(b) Quadrant 2

(c) Quadrant 3

(d) Quadrant 4

(e) $\frac{13\pi}{12}$ isn't a number, Dr. Ritter made it up

$$\frac{13\pi}{12} = \underbrace{\frac{12\pi}{12}}_{\pi} + \frac{\pi}{12}$$

Question

The angle $\frac{13\pi}{6}$ is coterminal with

$\frac{13\pi}{6} = 2\pi + \frac{\pi}{6}$

(a) $\frac{\pi}{6}$

(b) $\frac{5\pi}{6}$

(c) $\frac{7\pi}{6}$

(d) $\frac{11\pi}{6}$

(e) $\frac{13\pi}{6}$ isn't a number, Dr. Ritter made it up

Question

$$\sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos x}{2}}$$

The exact value of $\sin\left(\frac{13\pi}{12}\right) =$

(a) $\frac{1}{4}$

(b) $-\frac{\sqrt{3}}{4}$

(c) $-\frac{\sqrt{2-\sqrt{3}}}{2}$

(d) $\frac{\sqrt{2-\sqrt{3}}}{2}$