March 6 MATH 1112 sec. 54 Spring 2019

Section 6.3 Trigonometric Functions of any Angle

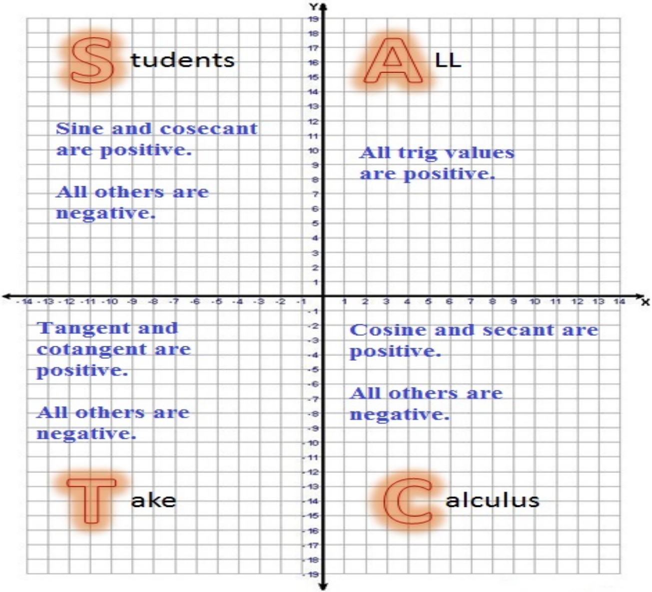
If θ is any angle in standard position, and (x, y) is any point on its terminal side (other than the origin) at a distance of $r = \sqrt{x^2 + y^2}$ from the origin, then the trigonometric values of θ are defined by

$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}$$

$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

Quadrants & Signs of Trig Values



Example

Determine which quadrant the terminal side of θ must be in if

(a) $\sin\theta > 0$ and $\tan\theta < 0$

$\sin\theta > 0 \Rightarrow \theta$ is in I or II

$\tan\theta < 0 \Rightarrow \theta$ is in II or IV

θ must be quadrant II

(b) $\sec\theta < 0$ and $\cot\theta > 0$

$\sec\theta < 0 \Rightarrow \theta$ is in II or III

$\cot\theta > 0 \Rightarrow \theta$ is in I or III

θ must be in quadrant III

Question

Suppose that θ is a positive angle whose measure is less than 360°, $\sin\theta = -0.3420$, and $\cos\theta = -0.9397$. Which of the following must be true about θ?

(a) $0^\circ < \theta < 90^\circ$

(b) $90^\circ < \theta < 180^\circ$

(c) $180^\circ < \theta < 270^\circ$

(d) $270^\circ < \theta < 360^\circ$

(e) any of the above may be true, more information is needed to determine which is true

$\sin\theta < 0$ and $\cos\theta < 0$

means θ has terminal side in quadrant $\underline{\text{III}}$

(in standard position)

Reference Angles

Suppose we want to find the trig values for the angle θ shown. Note that the acute angle (pink) has terminal side through (x, y), and by symmetry the terminal side of θ passes through the point $(-x, y)$ (same y and opposite sign x).

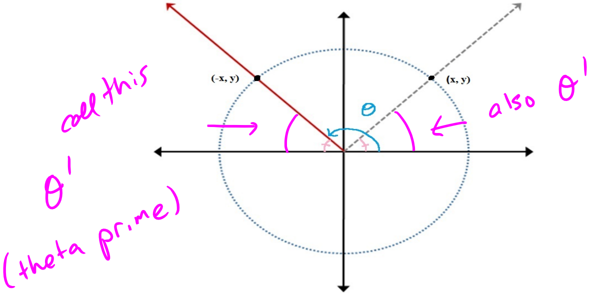
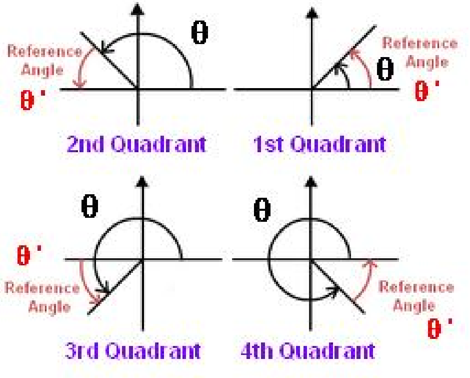


Figure: What is the connection between the trig values for θ and those for the acute angle in pink?

Reference Angles

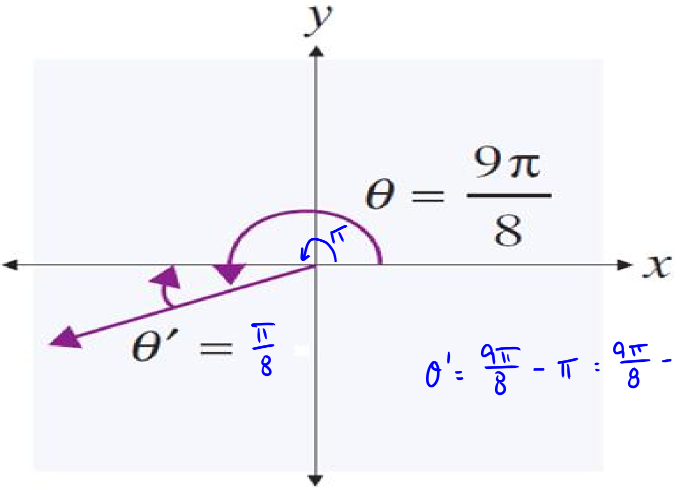
that is not quadrantal

Definition: Let θ be an angle in standard position. The **reference angle** θ' associated with θ is the angle of measure $0^\circ < \theta' < 90^\circ$ between the terminal side of θ and the *nearest* part of the x-axis.



Example (a)

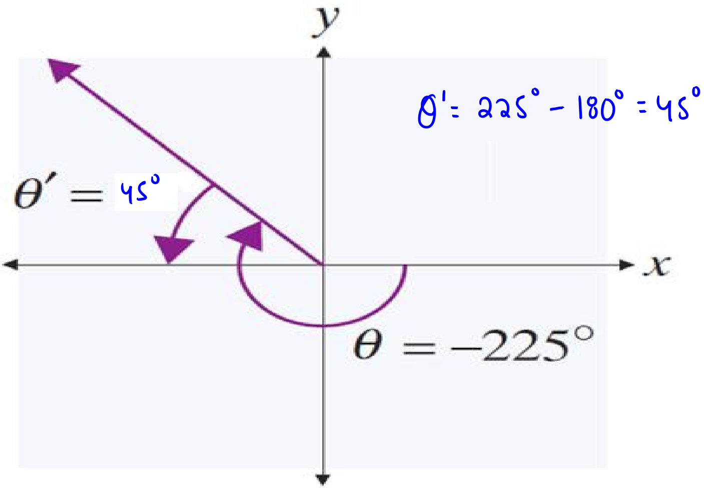
Determine the reference angle.



$\theta' = \frac{9\pi}{8} - \pi = \frac{9\pi}{8} - \frac{8\pi}{8} = \frac{\pi}{8}$

Example (b)

Determine the reference angle.



Question

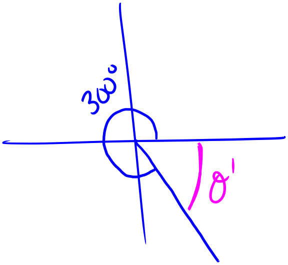
The reference angle for 300° is

(a) $-60°$

(b) $60°$

(c) $-30°$

(d) $30°$



$\theta' = 360° - 300°$

Theorem on Reference Angles

Theorem: If θ' is the reference angle for the angle θ, then

$$\sin\theta' = |\sin\theta|, \quad \cos\theta' = |\cos\theta| \quad \& \quad \tan\theta' = |\tan\theta|.$$

Remark 1: The analogous relationships hold for the cosecant, secant, and cotangent.

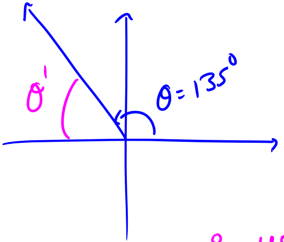
Remark 2: This means that the trigonometric values for θ can differ at most by a sign (+ or -) from the values for θ'.

This means that $\sin\theta = \sin\theta'$ or $\sin\theta = -\sin\theta'$

Example: Using Reference Angles

Find the exact value of

(a) $\sin(135^\circ)$



$\theta' = 180^\circ - 135^\circ = 45^\circ$

$\sin(135^\circ) = \sin(45^\circ)$

or

$\sin(135^\circ) = -\sin(45^\circ)$

$\sin(45^\circ) = \frac{\sqrt{2}}{2}$

$\sin\theta > 0$ in quad II

so $\boxed{\sin(135^\circ) = \frac{\sqrt{2}}{2}}$