March 8 MATH 1112 sec. 54 Spring 2019

Section 6.3 Trigonometric Functions of any Angle

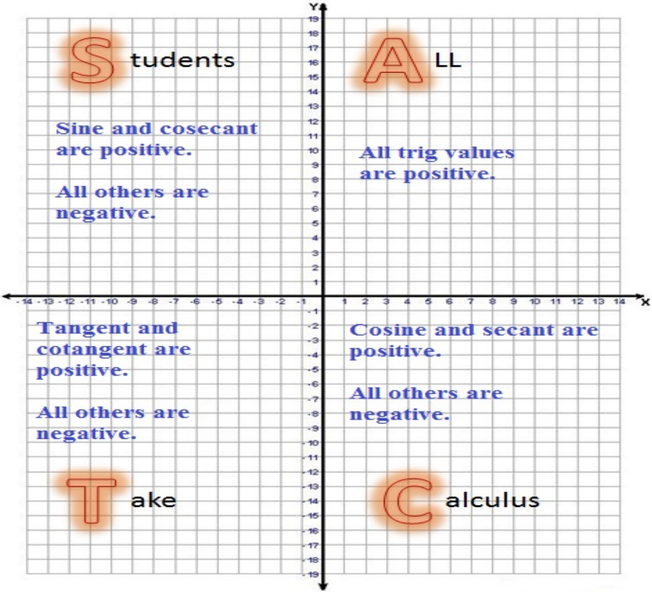
If θ is any angle in standard position, and (x, y) is any point on its terminal side (other than the origin) at a distance of $r = \sqrt{x^2 + y^2}$ from the origin, then the trigonometric values of θ are defined by

$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}$$

$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

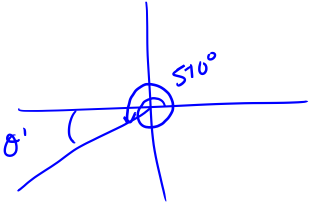
Quadrants & Signs of Trig Values



Example

Find the exact trigonometric value.

(a) $\sin(570^\circ)$



$570^\circ - 360^\circ = 210^\circ$

The reference angle

$\theta' = 210^\circ - 180^\circ$

$= 30^\circ$

$\sin 30^\circ = \frac{1}{2}$

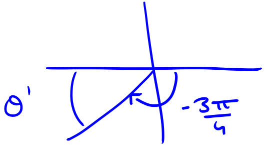
570° has terminal side in quadrant $\underline{\text{III}}$ where Sine < 0.

$\sin 570^\circ = -\sin 30^\circ = \frac{-1}{2}$

Example

Find the exact trigonometric value.

(b) $\cos\left(-\frac{3\pi}{4}\right)$



$\theta' = \pi - \frac{3\pi}{4} = \frac{\pi}{4}$

$\cos \pi/4 = \frac{1}{\sqrt{2}}$

$-\frac{3\pi}{4}$ is in quadrant III where cosine < 0

$\cos -\frac{3\pi}{4} = -\cos\frac{\pi}{4} = \frac{-1}{\sqrt{2}}$

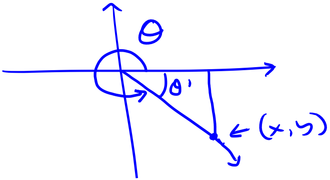
Example

Suppose that

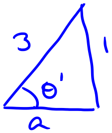
$$\sin\theta = -\frac{1}{3}$$

and when in standard position, the terminal side of θ is in the fourth quadrant. Determine the remaining trigonometric values of θ.

We'll draw a representative triangle.



$\sin\theta' = \frac{1}{3}$ $\quad \frac{\text{opp}}{\text{hyp}}$



$a^2+1^2=3^2 \Rightarrow a^2=8$

$a=2\sqrt{2}$

$x>0$ and $y<0$

so we can take $(x,y)=(2\sqrt{2},-1)$, $r=3$

$\sin\theta = \frac{-1}{3}$, $\cos\theta = \frac{2\sqrt{2}}{3}$

$\tan\theta = \frac{-1}{2\sqrt{2}}$, $\cot\theta = -2\sqrt{2}$

$\csc\theta = -3$, $\sec\theta = \frac{3}{2\sqrt{2}}$