May 31 Math 1190 sec. 51 Summer 2017

First Day of Class

Today we will complete a few activities. These include

- Introduction and syllabus highlights.
- A quick look at online resources.
- Syllabus and PreReq quiz info
- An introduction to clicker use.
- ...MATH

Introduction & Syllabus Highlights

- The Syllabus on Campus Server

- D2L

Online Resources

I'm a big fan of having a class webpage on the University server.

Our Class Webpage

Much of the same, and a few other resources will be available in D2L.

D2L

Right now in D2L, you can find

- The syllabus and prerequisite skills quizzes,
- Review material to help you strengthen algebra and trigonometry skills
- Course documents (syllabus/schedule)
- Helpful pencasts

Pencasts

Pencasts are pdf documents that contain an audio file. They can be viewed like a video on a computer or mobile device.

On a Mobile Device: Download the free LiveScribe+ app from the Apple Store or Google Play.

- In D2L, select ”download” from the drop down menu for the pencast you wish to see.
- When you select the file on your device, you should see an ”Open with...” option. Choose LiveScribe+
- Click play, or tap on the document to start viewing from a point other than the beginning.

Pencasts

On a Computer: Access the free viewer at www.livescribe.com (linked at the top of each pencast).

- In D2L, select ”download” from the drop down menu for the pencast you wish to see.
- Click the header of the pdf which will open the viewing page at livescribe.com.
- Drag and drop the pdf file (or select the choose file option). Click play, or click elsewhere on the document to begin play from a point other than the beginning.

Two Important Quizzes in D2L

There are two quizzes in D2L that you must complete. Each counts as one quiz grade.

(1) Please read the entire syllabus and take the Syllabus Quiz online in D2L. It must be completed online in D2L before 11:30 pm on Monday June 12.

(2) There is an Algebra and Trig Prerequisite Knowledge quiz. You are encouraged to take it **ASAP** and use it as a guide if you need to refresh your prereq. skills. You must take it prior to 11:30 pm on Sunday June 11. You have three attempts; the highest grade will be recorded. There is a 2 hour time limit per attempt.

Let's get started with Clickers

Throughout a typical class day, I will post questions for you to answer. Some will involve computation, others will be more conceptual.

The questions will appear in slides like this one, and will be either True/False or Multiple Choice.

You will be able to submit answers using our class set of Clickers.

Let's get started with Clickers

Each day you can grab a clicker and register it to my receiver. Your participation will be recorded. Two things to remember:

- **It is critical that you attend class and come on time to avoid missing out on clicker registration!**
- **I keep track of attendance with a sign in sheet, but clickers give me a second record for cross reference.**

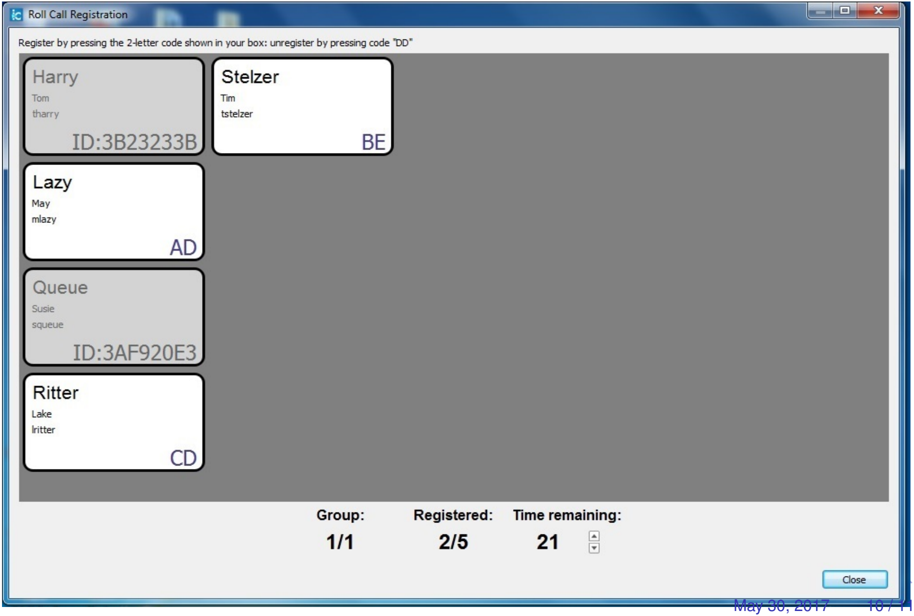
I want you to put in honest effort when answering clicker polls. **However** you are not being graded on your answers, so I don't want you to avoid answering when you're not certain.

Registering a Clicker

At the beginning of class, I will use the ”Roll Call” feature. You will see your name and student ID with a three letter code.

- Grab a clicker from my stash at the beginning of class.
- Look for your name with three letter code on the roll call display. (All names won’t fit on one screen, so it will alternate between groups.)
- Turn the clicker on, and methodically enter your three letter code.
- When your clicker is registered, your name box will turn gray with an ID code in the bottom right corner.
- If you press the wrong code, no worries, just press ”DD” (or ”DDD”).

Registering with Roll Call



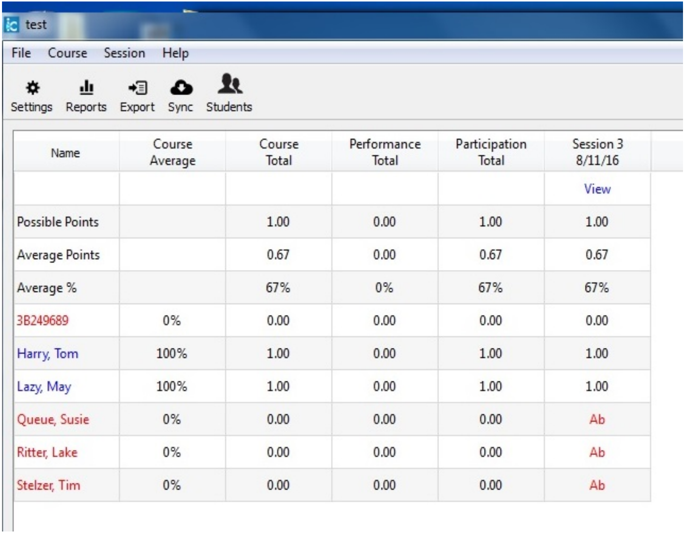
Help! I missed Roll Call Registration!

I got here late, **and I have a really good reason!**. Can I still participate in clicker polls?

- Grab a clicker, turn it on, and participate in remaining polls.
- Make note of the code on the back of the clicker you're using (e.g. "3D8903B7")
- **See me after class with this code, and make sure I register it to you.**

I expect very few after class clicker registrations.

Help! I missed Roll Call Registration!



Name	Course Average	Course Total	Performance Total	Participation Total	Session 3 8/11/16
					View
Possible Points		1.00	0.00	1.00	1.00
Average Points		0.67	0.00	0.67	0.67
Average %		67%	0%	67%	67%
3B249689	0%	0.00	0.00	0.00	0.00
Harry, Tom	100%	1.00	0.00	1.00	1.00
Lazy, May	100%	1.00	0.00	1.00	1.00
Queue, Susie	0%	0.00	0.00	0.00	Ab
Ritter, Lake	0%	0.00	0.00	0.00	Ab
Stelzer, Tim	0%	0.00	0.00	0.00	Ab

Figure: This is what I see when an unregistered clicker is used. I will only know it's you if you tell me.

Let's Try It Out

Sample Question 1

If you lived in a pineapple under the sea, what sort of invertebrate do you think you might be?

(a) An anemone
(b) A turtle
(c) A sponge
(d) A basket ball

Sample Question 2

For True/False questions, we'll always use "A" for true and "B" for false.

True/False The seasonal changes we experience through the year are due to the Earth's proximity to the Sun (closeness in orbit).

Sample Question 3

Let's try something math related

The expression $\tan\theta$ is equivalent to which of the following

(a) $\dfrac{\sin\theta}{\cos\theta}$

(b) $\dfrac{1}{\cot\theta}$

(c) $\cot\left(\dfrac{\pi}{2}-\theta\right)$

(d) All of the above.

Sample Question 4

True/False I feel reasonably confident that I'll get the hang of registering and using a clicker in this class.

Questions?

If you have any questions about the class structure, let's get them answered now.

Then we'll get started on our course!

Section 1.1: Limits of Functions Using Numerical and Graphical Techniques

In *Calculus*, we consider the way in which quantities **change.** In particular, if we have a function representing some process (motion of a particle, growth of a population, spread of a disease), we can analyze it to determine the nature of how it changes. We can also use knowledge of change to reconstruct a function describing a process.

Central to analyzing change and reconstructing functions is notion of a **limit.**

Slope

We know that a non-vertical line has the form

$$y = mx + b.$$

The slope m tells us how the dependent variable y will change if the independent variable x changes by a set amount Δx.

Slope

Consider $y = 3x - 1$ which has slope $m = 3$.

Note that two points on this line are $(1, 2)$ and $(3, 8)$. The change in x between these two points is

$$\Delta x = 3 - 1 = 2.$$

Compute the change in y, Δy.

$$\Delta y = 8 - 2 = 6 = 3 \cdot 2 = 3\,\Delta x = m\,\Delta x$$

$$\Delta y = m\,\Delta x \Rightarrow m = \frac{\Delta y}{\Delta x}$$

Question

The slope of the line containing the points $(2, 2)$ and $(3, 7)$ is

(a) $m = \frac{1}{5}$

(b) $m = 5$

(c) $m = \frac{7}{2}$

(d) can't be determined without more information

$\Delta y = 7 - 2 = 5$

$\Delta x = 3 - 2 = 1$

So $m = \frac{\Delta y}{\Delta x} = \frac{5}{1} = 5$

Slope of a general curve

So slope of a line tells us how y changes when x changes. What if the curve isn't a line??

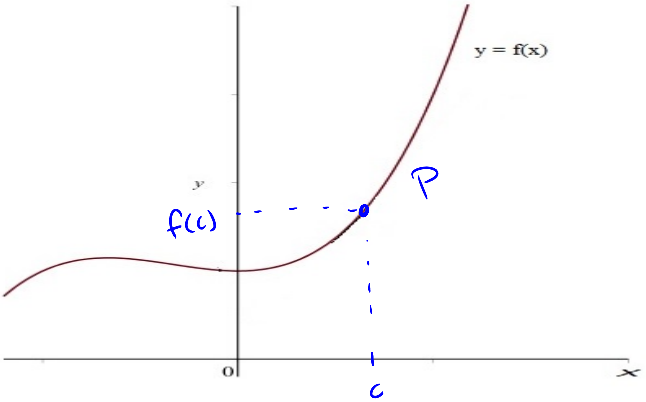


Figure: Can we consider *slope* for a curve like this?

The Tangent Line Problem

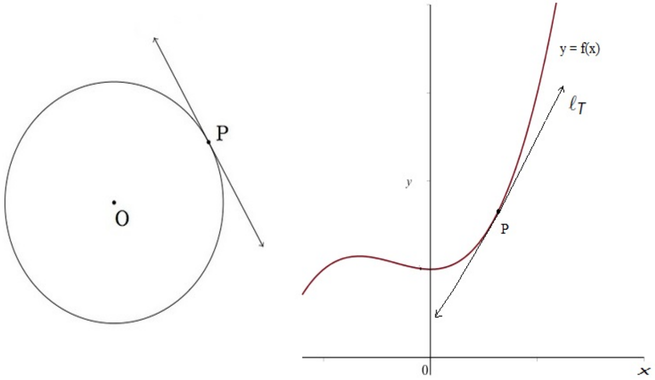


Figure: We begin by considering the tangent line problem. For a circle, a tangent line at point P is defined as the line having exactly one point in common with the circle. For the graph of a function $y = f(x)$, we define the tangent line at the point P has the line that shares the point P and has the same *slope* as the graph of f at P.

Slope of the Tangent Line

Question: What is meant by the *slope* of the function at the point P?

For now, let's assume that the graph if reasonably *nice* like the one in the figure. Let P be at $x = c$ and $y = f(c)$

$$\text{i.e.} \quad P = (c, f(c)).$$

To find a slope, we require two points. So let's take another point Q on the graph of f. In term of coordinates

$$Q = (x, f(x)).$$

The line through the two points P and Q on the graph is called a **Secant Line**. We will denote the slopes of the tangent line and the secant line as

$$m_{tan} \quad \text{and} \quad m_{sec}.$$

Slope of the Tangent Line

We have two points on our curve $y = f(x)$. The points at c and x (with $x \neq c$) are

$$P = (c, f(c)), \quad \text{and} \quad Q = (x, f(x)).$$

Compute the slope m_{sec} of the secant line through these points.

$m = \frac{\Delta y}{\Delta x}$ $\qquad$ $\Delta y = f(x) - f(c)$, $\quad \Delta x = x - c$

$$m_{sec} = \frac{f(x) - f(c)}{x - c}$$

Slope of the Tangent Line

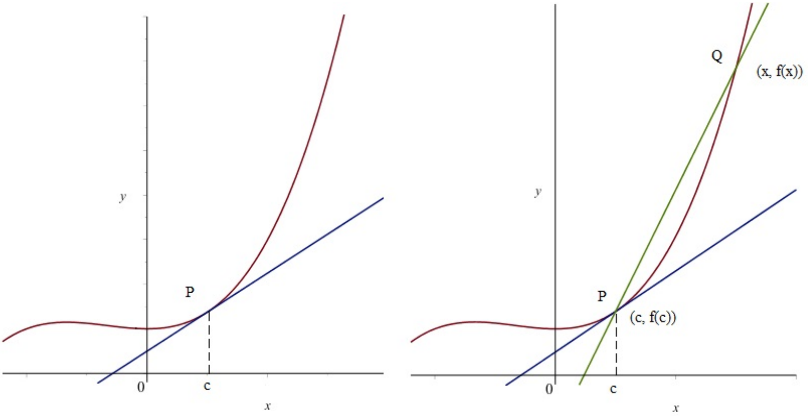
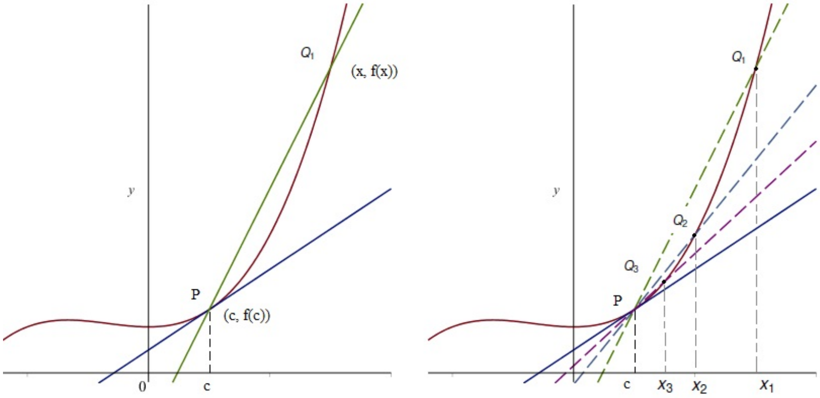


Figure: The slope of the line through P and Q (rise over run) is

$$m_{sec} = \frac{f(x) - f(c)}{x - c}$$

Slope of the Tangent Line

We consider a sequence of points $Q_1 = (x_1, f(x_1))$, $Q_2 = (x_2, f(x_2))$, and so forth in such a way that the x-values are getting closer to c. Note that the resulting secant lines tend to have slopes closer to that of the tangent line.



Slope of the Tangent Line

We call this process a *limit*. We will define the slope of the tangent line as

$$m_{tan} = \left[\text{Limit of } \quad \frac{f(x) - f(c)}{x - c} \quad \text{as } x \text{ gets closer to } c\right].$$

Our notation for this process will be

$$m_{tan} = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}.$$

The notation $\lim_{x \to c}$ reads as "the limit as x approaches c."

Notation: The notation $\lim_{x \to c}$ is always followed by an algebraic expression. It is never immediately followed by an equal sign.

A Working Definition of a Limit

Definition: Let f be defined on an open interval containing the number c except possibly at c. Then

$$\lim_{x \to c} f(x) = L$$

provided the value of $f(x)$ can be made arbitrarily close to the number L by taking x sufficiently close to c but not equal to c.

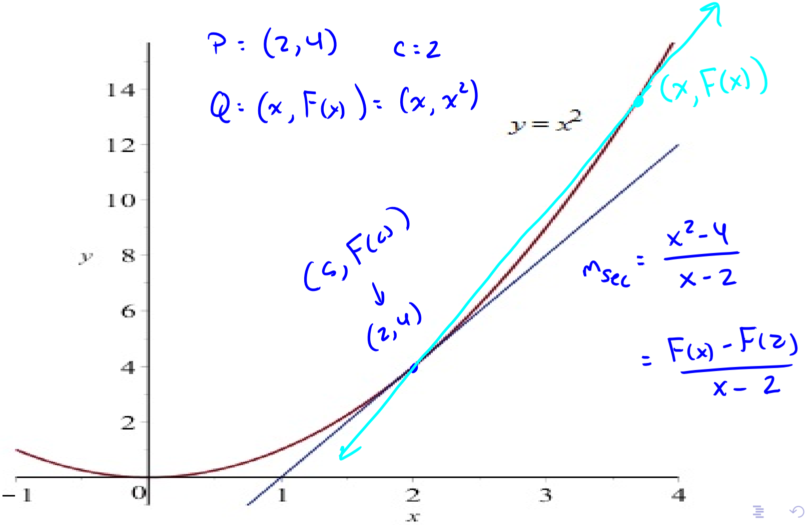
- f need not be defined @ c
- x = c is not included.

The number c is associated with x;

the number L is associated with y.

Example

Use a calculator to determine the slope of the line tangent to the graph of $F(x) = x^2$ at the point $(2, 4)$.



$$m_{sec} = \frac{x^2-4}{x-2}, \quad m_{tan} = \lim_{x\to 2} m_{sec} = \lim_{x \to 2} \frac{x^2-4}{x-2}$$

$$\frac{F(1.9)-F(2)}{1.9-2} = \frac{(1.9)^2-4}{1.9-2} = 3.9$$

$$m_{tan} = \lim_{x\to 2} \frac{x^2-4}{x-2}$$

$$= 4$$

x	$\frac{F(x)-F(2)}{x-2}$
1.9	3.9
1.99	3.99
1.999	3.999
2	undefined
2.001	4.001
2.01	4.01
2.1	4.1

looks like, they're going to 4

Example

Use a calculator and table of values to investigate

$$\lim_{x\to 0} \frac{e^x - 1}{x}$$

c=0 were taking x to 0.

x	$f(x) = \frac{e^x-1}{x}$
-0.1	0.9516
-0.01	0.9950
-0.001	0.9995
0	undefined
0.001	1.0005
0.01	1.0050
0.1	1.0517

$f(-0.1) = \frac{e^{-0.1} - 1}{-0.1}$

appear to tend to 1

$\lim_{x\to 0} \frac{e^x - 1}{x} = 1$

Question

True or False: In order to evaluate $\lim_{x \to c} f(x)$, the value of $f(c)$ must be defined (i.e. c must be in the domain of f)?

f needs to be defined near c, but not at c.

Left and Right Hand Limits

In our examples, we considered x-values to the left (less than) and to the right (greater than) c. This illustrates the notion of **one sided limits**. We have a special notation for this.

Left Hand Limit: We write

$$\lim_{x \to c^-} f(x) = L_L$$

and say *the limit as x approaches c from the left of $f(x)$ equals L_L provided we can make $f(x)$ arbitrarily close to the number L_L by taking x sufficiently close to, but less than c.*

Left and Right Hand Limits

Right Hand Limit: We write

$$\lim_{x\to c^+} f(x) = L_R$$

and say *the limit as x approaches c from the right of $f(x)$ equals L_R provided we can make $f(x)$ arbitrarily close to the number L_R by taking x sufficiently close to, but greater than c.*

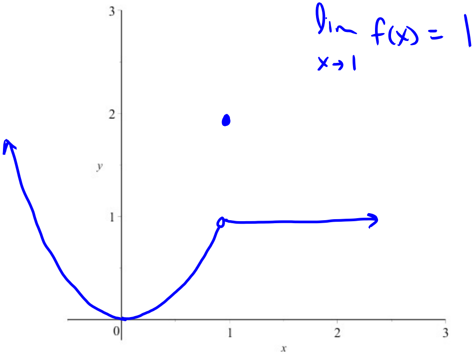
Some other common phrases:

"from the left" is the same as "from below"

"from the right" is the same as "from above."

Example

Plot the function $f(x) = \begin{cases} x^2, & x < 1 \\ 2, & x = 1 \\ 1, & x > 1 \end{cases}$ Investigate $\lim_{x \to 1} f(x)$ using the graph.

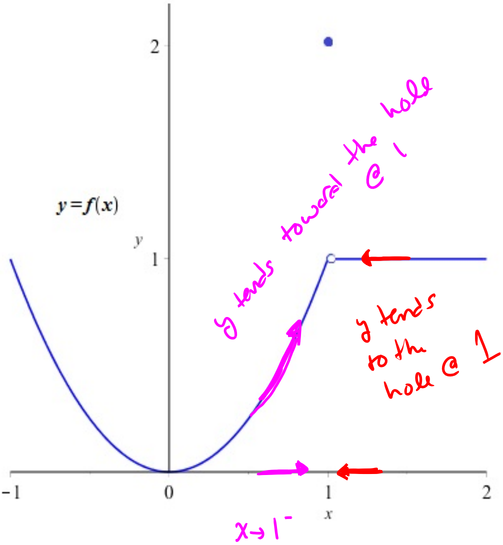


x	$f(x)$
0.9	$f(0.9) = (0.9)^2 = 0.81$
0.99	0.9801
0.999	0.9980
1	2
1.001	1
1.01	1
1.1	1

$\to 1$ (for $x = 0.9, 0.99, 0.999$)

$\to 1$ (for $x = 1.001, 1.01, 1.1$)

Example Continued...



$$\lim_{x \to 1} f(x) = 1$$

even though $f(1) = 2$

$$\lim_{x \to 1^-} f(x) = 1$$

$$\lim_{x \to 1^+} f(x) = 1$$

Observations

Observation 1: The limit L of a function $f(x)$ as x approaches c does not depend on whether $f(c)$ exists or what it's value may be.

Observation 2: If $\lim_{x\to c} f(x) = L$, then the number L is unique. That is, a function can not have two different limits as x approaches a single number c.

Observation 3: A function need not have a limit as x approaches c. If $f(x)$ can not be made arbitrarily close to any one number L as x approaches c, then we say that $\lim_{x\to c} f(x)$ **does not exist** (shorthand **DNE**).

Questions

(1) **True or False** It is possible that both $\lim_{x \to 3} f(x) = 5$ AND $f(3) = 7$.

In our last example $\lim_{x \to 1} f(x) = 1$ and $f(1) = 2$

(2) **True or False** It is possible that both $\lim_{x \to 3} f(x) = 5$ AND $\lim_{x \to 3} f(x) = 7$.

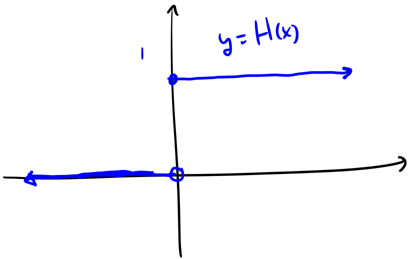
A limit is unique.

A Limit Failing to Exist

Consider $H(x) = \begin{cases} 0, & x < 0 \\ 1, & x \geq 0 \end{cases}$. Evaluate if possible

$$\lim_{x\to 0^-} H(x), \qquad \lim_{x\to 0^+} H(x), \qquad \text{and} \qquad \lim_{x\to 0} H(x)$$

H is called the Heaviside step function.



From the graph

$\lim_{x\to 0^-} H(x) = 0$

$\lim_{x\to 0^+} H(x) = 1$

We can say that $\lim_{x\to 0} H(x)$ doesn't exist.

The values that H approaches depends on the direction from which zero is approached.

H has an example of a "jump."

We can state the conclusion as

$$\lim_{x\to 0} H(x) \text{ DNE}$$

DNE = does not exist

Weakness of Technology

Suppose we wish to investigate

$$\lim_{x\to 0} \sin\left(\frac{\pi}{x^2}\right).$$

We consider values of x closer to zero, and plug them into a calculator. Let's look at two attempts.

x	$\sin\left(\frac{\pi}{x^2}\right)$
-0.1	0
-0.01	0
-0.001	0
0	undefined
0.001	0
0.01	0
0.1	0

limit looks like zero

x	$\sin\left(\frac{\pi}{x^2}\right)$
$-\frac{2}{3}$	0.707
$-\frac{2}{13}$	0.707
$-\frac{2}{23}$	0.707
0	undefined
$\frac{2}{23}$	0.707
$\frac{2}{13}$	0.707
$\frac{2}{3}$	0.707

limit looks like 0.707

Weakness of Technology

In every interval containing zero, the graph of $\sin(\pi/x^2)$ passes through every y-value between -1 and 1 infinitely many times.

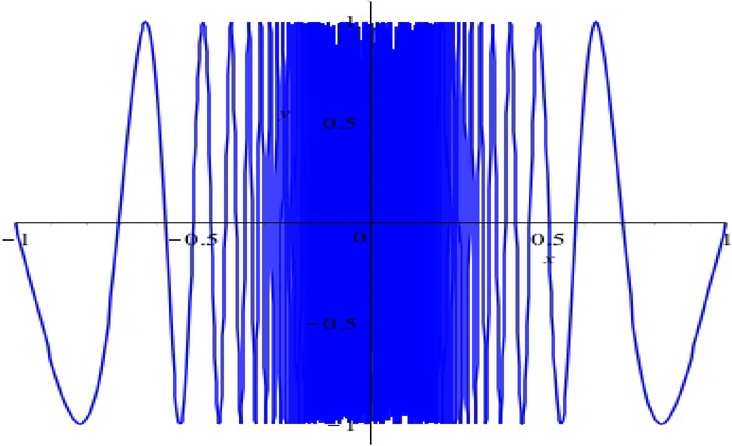


Figure: $y = \sin\left(\frac{\pi}{x^2}\right)$

Evaluating Limits

As this example illustrates, we would like to avoid too much reliance on technology for evaluating limits. The next section will be devoted to techniques for doing this for reasonably well behaved functions. We close with one theorem.

Theorem: Let f be defined on an open interval containing c except possible at c. Then

$$\lim_{x\to c} f(x) = L$$

if and only if

$$\lim_{x\to c^-} f(x) = L \quad \text{and} \quad \lim_{x\to c^+} f(x) = L.$$

Limits from a graph

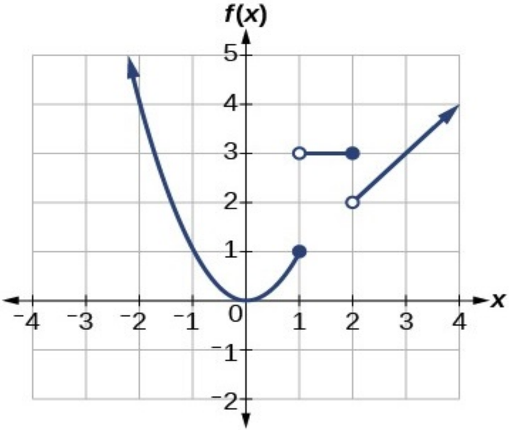
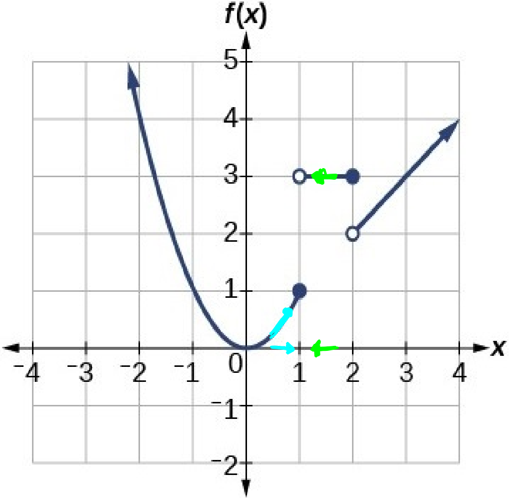


Figure: Given a graph $y = f(x)$, we may be able to evaluate limits visually. $\lim_{x \to c} f(x) = L$ would mean that the y values get closer to the number L when the x values get closer to the number c.

Evaluate if possible: $\lim_{x\to 1^-} f(x)$ and $\lim_{x\to 1^+} f(x)$



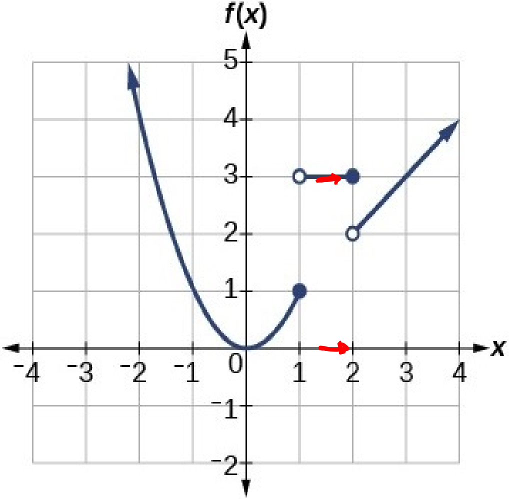
$\lim_{x\to 1^-} f(x) = 1$

$\lim_{x\to 1^+} f(x) = 3$

$\lim_{x\to 1} f(x)$ DNE

one sided limits don't agree

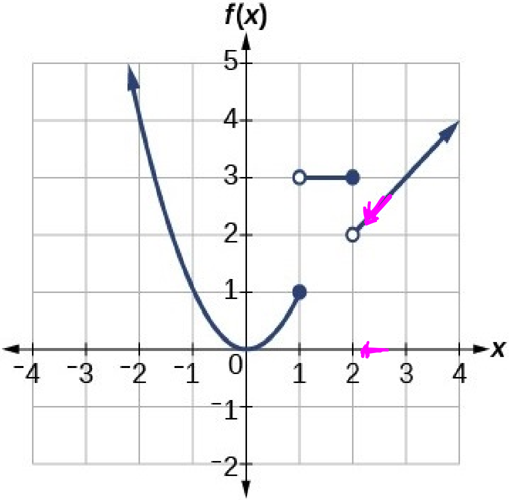
Question



$$\lim_{x \to 2^-} f(x) =$$

(a) 1 (b) 2 (c) 3

Question



$$\lim_{x \to 2^+} f(x) =$$

(a) 1 (b) 2 (c) 3

Section 1.2: Limits of Functions Using Properties of Limits

We begin with two of the simplest limits we may encounter.

Theorem: If $f(x) = A$ where A is a constant, then for any real number c

$$\lim_{x\to c} f(x) = \lim_{x\to c} A = A$$

The limit of a constant is that constant.

Theorem: If $f(x) = x$, then for any real number c

$$\lim_{x\to c} f(x) = \lim_{x\to c} x = c$$

Examples

(a) $\lim_{x\to 0} 7 =$ 7   limit of a constant

(b) $\lim_{x\to \pi} 3\pi =$ 3π ditto

(c) $\lim_{x\to -\sqrt{5}} x =$ $-\sqrt{5}$ limit of x

(d) $\lim_{x\to 4^-} x =$ 4

Because $\lim_{x\to 4} x = 4$ which implies both $\lim_{x\to 4^-} x = 4$ and $\lim_{x\to 4^+} x = 4$

Additional Limit Law Theorems

Suppose

$$\lim_{x\to c} f(x) = L, \quad \lim_{x\to c} g(x) = M, \quad \text{and } k \text{ is constant.}$$

Theorem: (Sums) $\lim_{x\to c}(f(x)+g(x)) = L+M$

Theorem: (Differences) $\lim_{x\to c}(f(x)-g(x)) = L-M$

Theorem: (Constant Multiples) $\lim_{x\to c} kf(x) = kL$

Theorem: (Products) $\lim_{x\to c} f(x)g(x) = LM$

Examples

Use the limit law theorems to evaluate if possible

(a) $\lim_{x\to 2}(3x+2)$ $= \lim_{x\to 2} 3x + \lim_{x\to 2} 2$ sum property

$= 3\lim_{x\to 2} x + \lim_{x\to 2} 2$ constant multiple property

$= 3(2) + 2$

$= 6 + 2 = 8$

Examples

Use the limit law theorems to evaluate if possible

(b) $\lim_{x\to -3}(x+1)^2 = \lim_{x\to -3}(x+1)(x+1) = \lim_{x\to -3}(x+1)\cdot\lim_{x\to -3}(x+1)$

$= -2(-2) = 4$

Note $\lim_{x\to -3}(x+1) = \lim_{x\to -3} x + \lim_{x\to -3} 1$

$= -3 + 1 = -2$

Examples

Use the limit law theorems to evaluate if possible

(c) $\lim_{x\to 0} f(x)$ where $f(x) = \begin{cases} x+2, & x<0 \\ 1, & x=0 \\ 2x-3, & x>0 \end{cases}$

we have to consider one sided limits

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} (x+2) = \lim_{x\to 0^-} x + \lim_{x\to 0^-} 2 = 0 + 2 = 2$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (2x-3) = \lim_{x\to 0^+} 2x - \lim_{x\to 0^+} 3$$

$$= 2\lim_{x\to 0^+} x - \lim_{x\to 0^+} 3 = 2\cdot 0 - 3 = -3$$

Turns out, $\lim_{x\to 0} f(x)$ DNE.

since the one sided limits are different

Question

(1) $\lim_{x \to 1} f(x)$ where $f(x) = \begin{cases} x^2 + 1, & x \leq 1 \\ 3 - x, & x > 1 \end{cases}$

(a) 4

(b) 2

(c) 1

(d) DNE

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (x^2+1) = 2$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (3-x) = 2$

$\lim_{x \to 1^-} (x^2+1) = \lim_{x \to 1^-} (x \cdot x + 1) = \lim_{x \to 1^-} x \cdot \lim_{x \to 1^-} x + \lim_{x \to 1^-} 1$

$= 1 \cdot 1 + 1 = 1 + 1 = 2$

Question

(2) $\lim_{x \to -1} f(x)$ where $f(x) = \begin{cases} x^2 + 1, & x \leq 1 \\ 3 - x, & x > 1 \end{cases}$

(a) 4

(b) 2

(c) 1

(d) DNE

Since $-1 < 1$

$\lim_{x \to -1} f(x) = \lim_{x \to -1} (x^2+1) = 2$

Additional Limit Law Theorems

Suppose $\lim_{x \to c} f(x) = L$ and n is a positive integer.

Theorem: (Power) $\lim_{x \to c} (f(x))^n = L^n$

Note in particular that this tells us that $\lim_{x \to c} x^n = c^n$.

Theorem: (Root) $\lim_{x \to c} \sqrt[n]{f(x)} = \sqrt[n]{L}$ (if this is defined)

Combining the sum, difference, constant multiple and power laws:
Theorem: If $P(x)$ is a polynomial, then

$$\lim_{x \to c} P(x) = P(c).$$

Question

(1) $\lim_{x \to 2}(3x^2 - 4x + 7) =$

(a) 7

(b) DNE

(c) -11

(d) 11

polynomial so just plug in 2

$\lim_{x \to 2}(3x^2 - 4x + 7) = 3(2)^2 - 4(2) + 7 = 11$

Notation Reminder

The notation "$\lim_{x \to c}$" is **always** followed by a function expression and never immediately by an equal sign.

for comparison, think about the sentence

" $\sqrt{}$ = 2 ".

Question

(2) Suppose that we have determined that $\lim_{x\to 7} f(x) = 13$.

True or False: It is acceptable to write this as

$$"\lim_{x\to 7} = 13"$$

This makes as much sense

as $\sqrt{} = 13$

Additional Limit Law Theorems

Suppose $\lim_{x \to c} f(x) = L, \quad \lim_{x \to c} g(x) = M$ and $M \neq 0$

Theorem: (Quotient) $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{L}{M}$

Combined with our result for polynomials:

Theorem: If $R(x) = \frac{p(x)}{q(x)}$ is a rational function, and c is in the domain of R, then

$$\lim_{x \to c} R(x) = R(c).$$

Examples

Evaluate $\lim_{x \to 2} \dfrac{x^2+5}{x^2+x-1}$

$R(x) = \dfrac{x^2+5}{x^2+x-1}$ is rational and $2^2+2-1 = 6-1 = 5 \neq 0$

so 2 is in the domain of R.

Hence $\lim_{x \to 2} \dfrac{x^2+5}{x^2+x-1} = \dfrac{2^2+5}{2^2+2-1} = \dfrac{9}{5}$

Examples

Evaluate $\lim_{x\to 1} \frac{\sqrt{x+1}}{x+5} = \frac{\sqrt{2}}{6}$

By the n^{th} root property $\lim_{x\to 1} \sqrt{x+1} = \sqrt{\lim_{x\to 1}(x+1)} = \sqrt{1+1}$

$= \sqrt{2}$

Also $\lim_{x\to 1}(x+5) = 1+5 = 6 \leftarrow$ not zero

looks like $\frac{L}{M}$ with $M \neq 0$

Additional Techniques: When direct laws fail

Evaluate if possible $\displaystyle\lim_{x\to 2} \frac{x^2 - x - 2}{x^2 - 4}$

If $R(x) = \dfrac{x^2-x-2}{x^2-4}$, then R is rational, but $2^2-4 = 4-4 = 0$, so 2 is not in its domain.

The limit may or may not exist, more work is needed.

Note $2^2-2-2 = 4-2-2 = 0$

This tells us that $x-2$ is a factor of both numerator and denominator.

Since we don't need $x=2$, we may be able to get rid of the common factor. * For $x \neq 2$ $\frac{x-2}{x-2} = 1$

$$\lim_{x \to 2} \frac{x^2-x-2}{x^2-4} = \lim_{x \to 2} \frac{(x-2)(x+1)}{(x-2)(x+2)}$$

$$= \lim_{x \to 2} \frac{x+1}{x+2} = \frac{2+1}{2+2} = \frac{3}{4}$$