November 28 MATH 1113 sec. 51 Fall 2018

Section 7.3: Verifying Identities

Verify the identity $\quad \sec 2\theta = \dfrac{\sec^2\theta}{2-\sec^2\theta}$

From the right

$$\frac{\sec^2\theta}{2-\sec^2\theta} = \frac{\frac{1}{\cos^2\theta}}{2-\frac{1}{\cos^2\theta}}$$

reciprocal ID

$$= \frac{\frac{1}{\cos^2\theta}}{\frac{2\cos^2\theta}{\cos^2\theta}-\frac{1}{\cos^2\theta}}$$

algebra

$$= \frac{\frac{1}{\cos^2\theta}}{\frac{2\cos^2\theta - 1}{\cos^2\theta}}$$ algebra

$$= \frac{1}{\cos^2\theta} \cdot \left(\frac{\cos^2\theta}{2\cos^2\theta - 1}\right)$$ algebra

$$= \frac{1}{2\cos^2\theta - 1}$$

$$= \frac{1}{\cos 2\theta}$$ Double angle ID

$$= \sec 2\theta$$ as expected

Section 7.5: Trigonometric Equations

In this section, we wish to consider **conditional** equations involving trigonometric functions. Our goal will be to find a solution set.

Some examples of trigonometric equations include

$$2\cos(x) - 1 = 0, \qquad \sin\theta\cos\theta + \sin\theta = 0, \qquad 2\tan^2 x - \tan x - 1 = 0,$$

$$\csc 2\theta = \sec 2\theta, \qquad \tan^2(3x) = 3, \quad \text{and so forth.}$$

We'll use trigonometric identities, our knowledge of some trig values, and inverse trigonometric functions as needed.

A Couple of Simple Examples

Find all possible solutions of the equation $2\cos(x) - 1 = 0$.

Let's try to find solutions in the interval $[0, 2\pi)$.

$2\cos(x) - 1 = 0$

$2\cos(x) = 1$

$\cos(x) = \frac{1}{2}$ one solution is $\frac{\pi}{3}$ (from memory)

For any x having $\frac{\pi}{3}$ as a reference angle

Cosine will be $\frac{1}{2}$ or $\frac{-1}{2}$.

$\cos x > 0$ for x in quadrant IV

Another solution is $\frac{5\pi}{3}$

we found two basic solutions

$$\frac{\pi}{3} \quad \text{and} \quad \frac{5\pi}{3}.$$

To capture the solutions due to periodicity, the solutions can be listed as

$$x = \frac{\pi}{3} + 2\pi n \quad \text{or}$$

$$x = \frac{5\pi}{3} + 2\pi n \quad \text{where } n \text{ is any integer.}$$

Graphical Representation

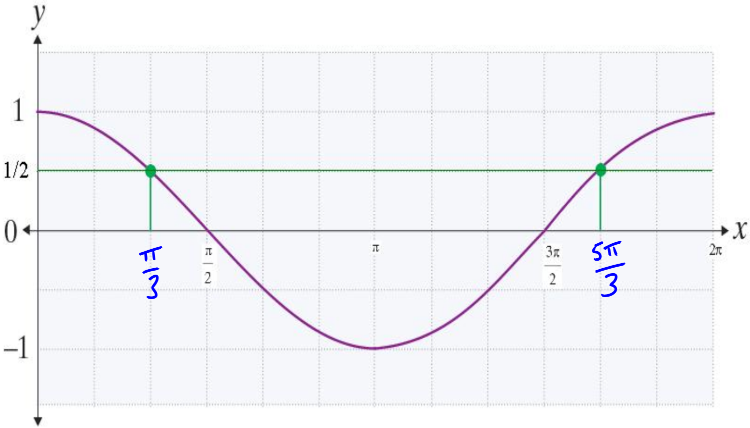

Figure: The solutions of $2\cos(x) - 1 = 0$ correspond to intersections of the curves $y = \cos x$ and $y = \frac{1}{2}$. Intersections continue to the left and right every 2π units.

Another Simple Example

Find all possible solutions of the equation $\sin(x) = \cos(x)$.

If $\cos x = 0$ then $\sin x = 1$ or $\sin x = -1$.

So $\cos x$ can't be zero.

$$\sin x = \cos x \implies \frac{\sin x}{\cos x} = 1$$

$\tan x = 1$ one solution is $\frac{\pi}{4}$

another is $\frac{5\pi}{4}$

All solutions are given by

$$x = \frac{\pi}{4} + 2\pi n \quad \text{or}$$

$$x = \frac{5\pi}{4} + 2\pi n \quad \text{for integers } n.$$

Graphical Representation

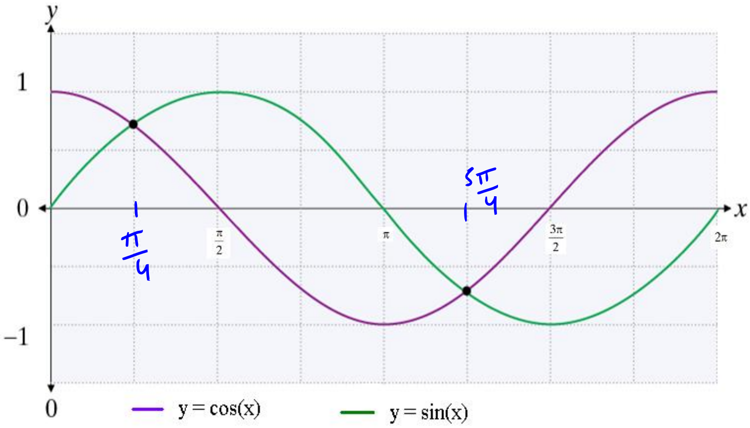

Figure: The solutions of $\sin(x) = \cos(x)$ correspond to intersections of the curves $y = \cos x$ and $y = \sin(x)$. Intersections continue to the left and right every 2π units.

A General Observation

When solving more complicated trigonometric equations, we will try to rewrite the problem in the form of **one or more** equations that look like

$$\text{One Trig Function} \quad = \quad \text{One Number}$$

We typically determine solution(s) in one period, and then extend those solutions if required.

Example

Find all possible solutions of the equation $1 + \cos\theta = 2\sin^2\theta$.

It's best to get all in terms of sine or cosine.

Use $\sin^2\theta = 1 - \cos^2\theta$

$$1 + \cos\theta = 2\sin^2\theta$$

$$1 + \cos\theta = 2(1 - \cos^2\theta)$$

$$1 + \cos\theta = 2 - 2\cos^2\theta$$

$$2\cos^2\theta + \cos\theta - 1 = 0$$

$$(2\cos\theta - 1)(\cos\theta + 1) = 0$$

Looks like

$$2u^2 + u - 1 = 0$$

$$(2u - 1)(u + 1) = 0$$

Using the zero product property

$2\cos\theta - 1 = 0$ or $\cos\theta + 1 = 0$

$\cos\theta = \frac{1}{2}$ or $\cos\theta = -1$

One solution is

$\theta = \frac{\pi}{3} + 2\pi n$

$\theta = \pi$

or

All solutions are

$\theta = \frac{5\pi}{3} + 2\pi n$

$\theta = \pi + 2\pi n$ for integers n.

from earlier

The solution set is

$$\theta = \frac{\pi}{3} + 2n\pi \quad \text{or} \quad \theta = \frac{5\pi}{3} + 2\pi n \quad \text{or}$$

$$\theta = \pi + 2\pi n$$

for integers n.