November 28 MATH 1113 sec. 52 Fall 2018

Section 7.3: Verifying Identities

Verify the identity $\sec 2\theta = \dfrac{\sec^2\theta}{2-\sec^2\theta}$

From the right

$$\frac{\sec^2\theta}{2-\sec^2\theta} = \frac{\frac{1}{\cos^2\theta}}{2-\frac{1}{\cos^2\theta}}$$

reciprocal ID

$$= \frac{\frac{1}{\cos^2\theta}}{\frac{2\cos^2\theta}{\cos^2\theta}-\frac{1}{\cos^2\theta}}$$

algebra

$$= \frac{\frac{1}{\cos^2\theta}}{\frac{2\cos^2\theta - 1}{\cos^2\theta}}$$ algebra

$$= \frac{1}{\cos^2\theta} \cdot \left(\frac{\cos^2\theta}{2\cos^2\theta - 1}\right)$$ algebra

$$= \frac{1}{2\cos^2\theta - 1}$$ algebra

$$= \frac{1}{\cos 2\theta}$$ Double angle ID

$$= \sec 2\theta$$ as expected.

Section 7.5: Trigonometric Equations

In this section, we wish to consider **conditional** equations involving trigonometric functions. Our goal will be to find a solution set.

Some examples of trigonometric equations include

$$2\cos(x) - 1 = 0, \qquad \sin\theta\cos\theta + \sin\theta = 0, \qquad 2\tan^2 x - \tan x - 1 = 0,$$

$$\csc 2\theta = \sec 2\theta, \qquad \tan^2(3x) = 3, \quad \text{and so forth.}$$

We'll use trigonometric identities, our knowledge of some trig values, and inverse trigonometric functions as needed.

A Couple of Simple Examples

Find all possible solutions of the equation $2\cos(x) - 1 = 0$.

We'll look for solutions on the interval $[0, 2\pi)$

then extend that.

$2\cos x - 1 = 0$

$2\cos x = 1$

$\cos x = \frac{1}{2}$ one solution is $\frac{\pi}{3}$ (from memory)

For any x having $\frac{\pi}{3}$ as its reference angle

$\cos x = \frac{1}{2}$ or $\cos x = \frac{-1}{2}$

$\cos x > 0$ in quad $\overline{\text{IV}}$

Another solution is $\frac{5\pi}{3}$.

Due to periodicity we can add any integer multiple of 2π to get other solutions.

The solution set can be expressed as

$$\theta = \frac{\pi}{3} + 2\pi n \quad \text{or}$$

$$\theta = \frac{5\pi}{3} + 2\pi n \quad \text{for integers } n$$

Graphical Representation

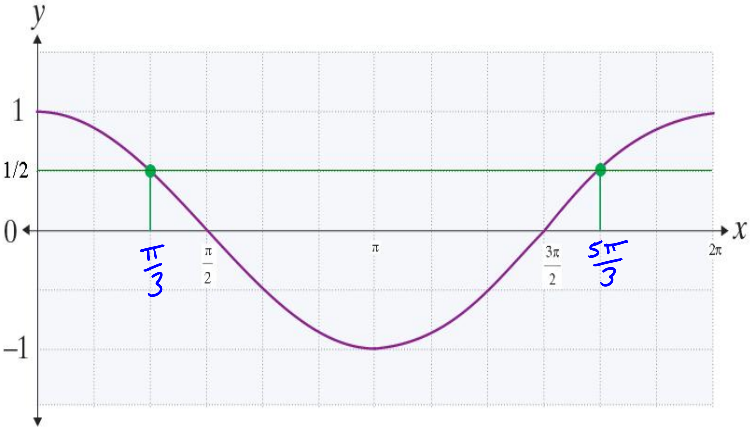

Figure: The solutions of $2\cos(x) - 1 = 0$ correspond to intersections of the curves $y = \cos x$ and $y = \frac{1}{2}$. Intersections continue to the left and right every 2π units.

Another Simple Example

Find all possible solutions of the equation $\sin(x) = \cos(x)$.

If $\cos x = 0$, then $\sin x = 1$ or $\sin x = -1$. So

$\cos x \neq 0$ for any solutions.

Divide by $\cos x$

$$\frac{\sin x}{\cos x} = 1 \Rightarrow \tan x = 1$$

One solution is $\frac{\pi}{4}$

The period of tangent is π

So all solutions are given by

$$x = \frac{\pi}{4} + \pi n \quad \text{for integers } n$$

Since tangent is positive in Quad III

$\tan x = 1$ if $x = \frac{5\pi}{4}$

We could also state the solution set as

$$x = \frac{\pi}{4} + 2\pi n \quad \text{or}$$

$$x = \frac{5\pi}{4} + 2\pi n \quad \text{for integers } n$$

Graphical Representation

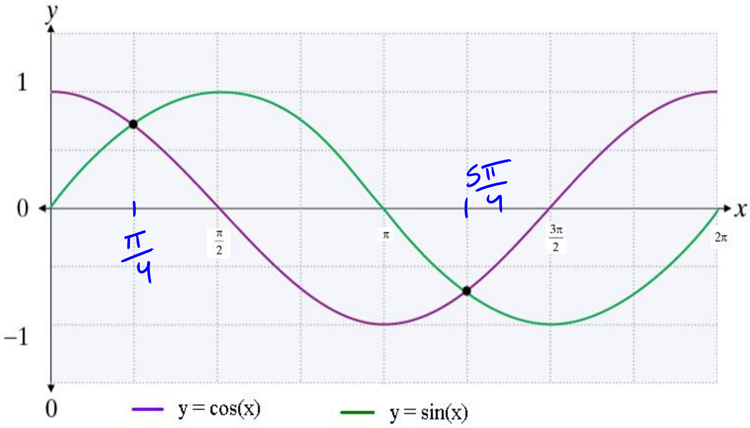

Figure: The solutions of $\sin(x) = \cos(x)$ correspond to intersections of the curves $y = \cos x$ and $y = \sin(x)$. Intersections continue to the left and right every 2π units.

A General Observation

When solving more complicated trigonometric equations, we will try to rewrite the problem in the form of **one or more** equations that look like

$$\text{One Trig Function} \quad = \quad \text{One Number}$$

We typically determine solution(s) in one period, and then extend those solutions if required.