November 2 Math 2306 sec 51 Fall 2015

Section 7.3: Translation Theorems

Theorem (translation in s) Suppose $\mathscr{L}\{f(t)\} = F(s)$. Then for any real number a

$$\mathscr{L}\left\{e^{at}f(t)\right\} = F(s-a).$$

Consequently $\mathscr{L}^{-1}\{F(s-a)\} = e^{at}f(t)$ if $\mathscr{L}^{-1}\{F(s)\} = f(t)$.

Solve the IVP using the Laplace Transform

$\mathscr{L}\{y(t)\} = Y(s)$

$y'' - 4y' + 13y = 39 \quad y(0) = 3, y'(0) = 1$

$$\mathscr{L}\{y'' - 4y' + 13y\} = \mathscr{L}\{39\}$$

$$\mathscr{L}\{y''\} - 4\mathscr{L}\{y'\} + 13\mathscr{L}\{y\} = 39\mathscr{L}\{1\}$$

$$s^2Y(s) - sy(0) - y'(0) - 4\left(sY(s) - y(0)\right) + 13Y(s) = \frac{39}{s}$$

$$(s^2 - 4s + 13)Y(s) - 3s - 1 + 12 = \frac{39}{s}$$

$$(s^2-4s+13)Y(s) = \frac{39}{s} + 3s-11$$

$$Y(s) = \frac{39}{s(s^2-4s+13)} + \frac{3s-11}{s^2-4s+13}$$

Do partial fraction decomp(s)

$$\frac{39}{s(s^2-4s+13)} = \frac{As+B}{s^2-4s+13} + \frac{C}{s}$$

Clear fractions

$$39 = (As+B)s + C(s^2-4s+13)$$

$$\underline{0}s^2 + \underline{0}s + \underline{39} = As^2 + Bs + Cs^2 - 4Cs + 13C$$

$$= \underline{\underline{(A+C)}}s^2 + \underline{\underline{(B-4C)}}s + \underline{13}C$$

$$13C = 39 \Rightarrow C = 3$$

$$B - 4C = 0 \Rightarrow B = 4C = 12$$

$$A + C = 0 \Rightarrow A = -C = -3$$

$$Y(s) = \frac{-3s+12}{s^2-4s+13} + \frac{3}{s} + \frac{3s-11}{s^2-4s+13}$$

$$Y(s) = \frac{1}{s^2-4s+13} + \frac{3}{s}$$

complete the square

$$s^2-4s+13 = s^2-4s+4-4+13$$

$$= (s-2)^2+9$$

$$Y(s) = \frac{1}{(s-2)^2+9} + \frac{3}{s}$$

$$Y(s) = \frac{1}{3}\,\frac{3}{(s-2)^2+9} + 3\,\frac{1}{s}$$

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = \frac{1}{3}\mathcal{L}^{-1}\left\{\frac{3}{(s-2)^2+9}\right\} + 3\,\mathcal{L}^{-1}\left\{\frac{1}{s}\right\}$$

$$y = \frac{1}{3}e^{2t}\sin 3t + 3$$