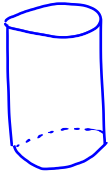
Nov. 4 Math 1190 sec. 51 Fall 2016

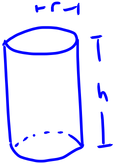
Section 4.7: Optimization

Let's Do One Together A can in the shape of a right circular cylinder is to have a volume of 128π cubic cm. The material that the top and bottom are made of costs \$0.20/cm^2 and the material that the lateral surface is made of costs \$0.10/cm^2. Find the dimensions of the can that minimize the total cost of production.



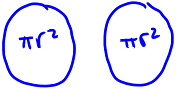
- We have a preset volume of 128π cm^3
- We want to minimize cost, so we need a cost function.

The surface of the con



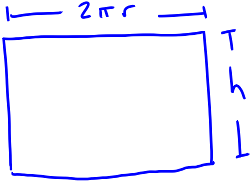
Let r be the base radius and h the height.

Top + Bottom



circles

Lateral surface



Top + Bottom area $= 2\pi r^2$

Lateral surface area $= 2\pi r h$

Total surface area

$$S = 2\pi r h + 2\pi r^2$$

Question

The total cost $C =$ (cost of lateral surface) + (cost of top & bottom). The cost for the lateral surface was \$0.10/cm^2 while the cost for the top and bottom material is \$0.20/cm^2. The surface area was $S = 2\pi rh + 2\pi r^2$. Which of the following is the cost function?

(a) $C = 2\pi rh + 2\pi r^2$

Cost / cm^2 · Area

(b) $C = 0.1(2\pi rh) + 0.2(2\pi r^2)$

(c) $C = 0.2(2\pi rh) + 0.1(2\pi r^2)$

(d) $C = (0.1)(0.2)(2\pi rh + 2\pi r^2)$

Question

The cost appears as a function of two variables, r and h. But we need it to be a function of only one variable.

The volume of the can $V = \pi r^2 h$. We are told it must hold 128π cm^3. Which of the following could be used to express C as a function of r alone?

$V = \pi r^2 h = 128\pi$

$r^2 h = 128 \Rightarrow h = \dfrac{128}{r^2}$

(a) $h = \dfrac{128}{r}$

(b) $r = \dfrac{128}{\sqrt{h}}$

(c) $h = \dfrac{128}{r^2}$

Question

We can write the cost function in terms of r as

$$C = \frac{25.6\pi}{r} + 0.4\pi r^2$$

Which of the following is the derivative of C with respect to r?

$C: 25.6\pi r^{-1} + 0.4\pi r^2$

(a) $\dfrac{dC}{dr} = \dfrac{-25.6\pi}{r^2} + 0.8\pi r$

$\dfrac{dC}{dr} = -25.6\pi r^{-2} + 2(0.4)\pi r$

$= \dfrac{-25.6\pi}{r^2} + 0.8\pi r$

(b) $\dfrac{dC}{dr} = \dfrac{-25.6\pi + 0.8\pi r}{r^2}$

(c) $\dfrac{dC}{dr} = \dfrac{-25.6\pi}{r^2} + 0.4\pi r$

Question

Given that $$\frac{dC}{dr} = \frac{-25.6\pi}{r^2} + 0.8\pi r,$$

The critical number(s) of C are

$C'(r) = 0 \Rightarrow$

$\frac{-25.6\pi}{r^2} + 0.8\pi r = 0 \Rightarrow \frac{25.6\pi}{r^2} = 0.8\pi r$

$\Rightarrow r^3 = \frac{25.6}{0.8} = 32$

$r = (32)^{1/3}$

(a) 0 and 32

(b) 0 and $\sqrt[3]{32}$

(c) can't be determined without more information

(d) $\sqrt[3]{32}$

Question

We suspect that the optimal size for the radius, the one that minimizes cost is $\sqrt[3]{32}$. We decide to use the second derivative test to check. We find that

$$\frac{d^2C}{dr^2} = \frac{d}{dr}\left(\frac{-25.6\pi}{r^2} + 0.8\pi r\right) = \frac{51.2\pi}{r^3} + 0.8\pi$$

With no computation, we determine that $r = \sqrt[3]{32}$ is a local minimum because

(a) $C''(r)$ is positive for all positive r, so the graph is concave up.

(b) $C''(r)$ is negative for all positive r, so the graph is concave up.

(c) $C''(r)$ is positive for all positive r, so the graph is concave down.

(d) $C''(r)$ is negative for all positive r, so the graph is concave down.

Question

Since the optimal $r = \sqrt[3]{32}$ and $h = 128/r^2$ our recommendation for minimizing the cost is a can with dimensions

(a) radius of $\sqrt[3]{32}$ cm and height $128/\sqrt[3]{32}$ cm

(b) radius of $\sqrt[3]{32}$ cm and height $128/\sqrt[3]{32^2}$ cm

(c) radius of $\sqrt[3]{32}$ cm and height 4 cm