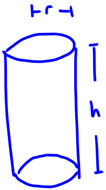
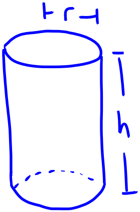
Nov. 4 Math 1190 sec. 52 Fall 2016

Section 4.7: Optimization

Let's Do One Together A can in the shape of a right circular cylinder is to have a volume of 128π cubic cm. The material that the top and bottom are made of costs $\$0.20/\text{cm}^2$ and the material that the lateral surface is made of costs $\$0.10/\text{cm}^2$. Find the dimensions of the can that minimize the total cost of production.

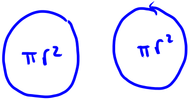


- The volume is preset @ 128π cm^3
- we want to minimize cost, so we need to construct a cost formula.

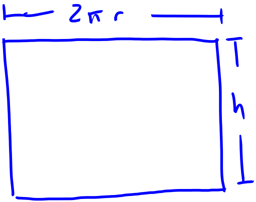


Let r and h be the radius and height of our cylinder, respectively.

Top + Bottom



Total area of Top + Bottom $= 2\pi r^2$



Total lateral surface area $= 2\pi r h$

Surface area of can

$$S = 2\pi r h + 2\pi r^2$$

Question

The total cost $C =$ (cost of lateral surface) + (cost of top & bottom). The cost for the lateral surface was \$0.10/cm^2 while the cost for the top and bottom material is \$0.20/cm^2. The surface area was $S = 2\pi rh + 2\pi r^2$. Which of the following is the cost function?

Cost = Cost / area · area

(a) $C = 2\pi rh + 2\pi r^2$

(b) $C = 0.1(2\pi rh) + 0.2(2\pi r^2)$

(c) $C = 0.2(2\pi rh) + 0.1(2\pi r^2)$

(d) $C = (0.1)(0.2)(2\pi rh + 2\pi r^2)$

Question

The cost appears as a function of two variables, r and h. But we need it to be a function of only one variable.

The volume of the can $V = \pi r^2 h$. We are told it must hold 128π cm^3. Which of the following could be used to express C as a function of r alone?

$V = \pi r^2 h = 128\pi$

$\pi r^2 h = 128\pi$

$r^2 h = 128$

$h = \frac{128}{r^2}$

(a) $h = \frac{128}{r}$

(b) $r = \frac{128}{\sqrt{h}}$

(c) $h = \frac{128}{r^2}$

Question

We can write the cost function in terms of r as

$$C = \frac{25.6\pi}{r} + 0.4\pi r^2$$

Which of the following is the derivative of C with respect to r?

(a) $\frac{dC}{dr} = \frac{-25.6\pi}{r^2} + 0.8\pi r$

(b) $\frac{dC}{dr} = \frac{-25.6\pi + 0.8\pi r}{r^2}$

(c) $\frac{dC}{dr} = \frac{-25.6\pi}{r^2} + 0.4\pi r$

$C = 25.6\pi r^{-1} + 0.4\pi r^2$

$\frac{dC}{dr} = -25.6\pi r^{-2} + 2(0.4)\pi r$

$= \frac{-25.6\pi}{r^2} + 0.8\pi r$

Question

Given that $$\frac{dC}{dr} = \frac{-25.6\pi}{r^2} + 0.8\pi r,$$

The critical number(s) of C are

zero is not in the domain of C

(a) 0 and 32

$C'(r) = 0 \Rightarrow \frac{-25.6\pi}{r^2} + 0.8\pi r = 0$

(b) 0 and $\sqrt[3]{32}$

$0.8\pi r = \frac{25.6\pi}{r^2}$

(c) can't be determined without more information

$r^3 = \frac{25.6}{0.8} = 32$

(d) $\sqrt[3]{32}$

$r = (32)^{1/3}$

Question

We suspect that the optimal size for the radius, the one that minimizes cost is $\sqrt[3]{32}$. We decide to use the second derivative test to check. We find that

$$\frac{d^2C}{dr^2} = \frac{d}{dr}\left(\frac{-25.6\pi}{r^2} + 0.8\pi r\right) = \frac{51.2\pi}{r^3} + 0.8\pi$$

With no computation, we determine that $r = \sqrt[3]{32}$ is a local minimum because

(a) $C''(r)$ is positive for all positive r, so the graph is concave up.

(b) $C''(r)$ is negative for all positive r, so the graph is concave up.

(c) $C''(r)$ is positive for all positive r, so the graph is concave down.

(d) $C''(r)$ is negative for all positive r, so the graph is concave down.

Question

Since the optimal $r = \sqrt[3]{32}$ and $h = 128/r^2$ our recommendation for minimizing the cost is a can with dimensions

(a) radius of $\sqrt[3]{32}$ cm and height $128/\sqrt[3]{32}$ cm

(b) radius of $\sqrt[3]{32}$ cm and height $128/\sqrt[3]{32^2}$ cm

(c) radius of $\sqrt[3]{32}$ cm and height 4 cm

Section 4.8: Antiderivatives; Differential Equations

Definition: A function F is called an antiderivative of f on an interval I if

$$F'(x) = f(x) \quad \text{for all } x \text{ in } I.$$

For example, $F(x) = x^2$ is an antiderivative of $f(x) = 2x$ on $(-\infty, \infty)$. Similarly, $G(x) = \tan x + 7$ is an antiderivative of $g(x) = \sec^2 x$ on $(-\pi/2, \pi/2)$.

Theorem: If F is any antiderivative of f on an interval I, then the *most general* antiderivative of f on I is

$$F(x) + C \quad \text{where } C \text{ is an arbitrary constant.}$$

Find the most general antiderivative of f.

(i) $f(x) = \cos x \quad I = (-\infty, \infty)$

$F(x) = \sin x$

So the most general is

$\sin x + C$

C is arbitrary

(ii) $f(x) = \dfrac{1}{x} \quad I = (0, \infty)$

$F(x) = \ln x$

most general is $\ln x + C$

Question: Find the most general antiderivative of f.

(iii) $f(x) = \dfrac{1}{1+x^2} \quad I = (-\infty, \infty)$

(a) $F(x) = \dfrac{x}{x + x^3/3} + C$

Note $\dfrac{d}{dx} \ln(x^2+1) = \dfrac{2x}{x^2+1}$

(b) $F(x) = \ln(1+x^2) + C$

$\dfrac{d}{dx} \tan^{-1} x = \dfrac{1}{1+x^2}$

(c) $F(x) = \tan^{-1} x + C$

(iv) $f(x) = \sec x \tan x \quad I = \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$

(a) $F(x) = \sec^2 x + C$

Since $\dfrac{d}{dx} \sec x = \sec x \tan x$

(b) $F(x) = \sec x + C$

(c) $F(x) = \tan x + C$