November 5 MATH 1113 sec. 51 Fall 2018

Section 6.4: Radian Measure

Recall that one *radian* is the measure of a central that subtends an arclength of one in a unit circle.

Converting Between Degrees & Radians

Since $360^\circ = 2\pi$ rad, we get the following conversion factors:

$$1^\circ = \frac{\pi}{180}\text{ rad} \qquad \text{and} \qquad 1\text{ rad} = \left(\frac{180}{\pi}\right)^\circ$$

Remark: If an angle doesn't have the degree symbol $^\circ$ next to it, it is assumed to be in radians!

Question

If θ is an angle of $-90°$, then in radians

(a) $\theta = \dfrac{\pi}{2}$

$$-90 \cdot \frac{\pi}{180} = \frac{-\pi}{2}$$

(b) $\theta = -\pi$

(c) $\theta = -\dfrac{2}{\pi}$

(d) $\theta = -\dfrac{\pi}{2}$

(e) there's no such thing as a negative angle

Question

If θ is an angle of measure 2, then

$\left(2 \cdot \frac{180}{\pi}\right)^{\circ} = \frac{360}{\pi}^{\circ}$

(a) $\theta = \dfrac{\pi}{90}$

(b) $\theta = \dfrac{360}{\pi}^{\circ}$

(c) $\theta = \dfrac{360}{\pi}$

(d) $\theta = \dfrac{\pi}{90}^{\circ}$

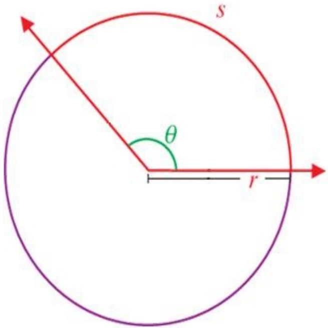
(e) there's no way to know if the given measure is degrees or radians

Arclength Formula

Given a circle of radius r, the length s of the arc subtended by the (positive) central angle θ (**in radians**) is given by

$$s = r\theta.$$

The area of the resulting sector is $A_{sector} = \frac{1}{2}r^2\theta$.



Motion on a Circle: Angular & Linear Speed

Definition: (angular speed) If an object moves along the arc of a circle through a central angle θ in the time t, the angular speed is denoted by ω (lower case omega) and is defined by

$$\omega = \frac{\theta}{t} = \frac{\text{angle moved through}}{\text{time}}.$$

θ in radians

Definition: (linear speed) If the circle has radius r, then the distance traveled is the arclength $s = r\theta$. The linear speed is denoted by ν (lower case nu) and is defined by

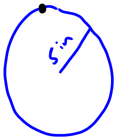
$$\nu = \frac{s}{t} = \frac{r\theta}{t} = r\omega.$$

Note that this is distance (s) per unit time (t).

Example

A flywheel with a 5 inch radius is rotating at a rate of 2 rotations per minute. What is the linear speed of a point on the rim of the wheel in inches per minute? How far does that point on the rim travel in 30 seconds?

$$v = \frac{s}{t} = \frac{r\theta}{t} = r\omega$$



We're given ω as 2 rotations per minute. We convert this to radians per minute

1 rotation = 2π radians

So 2 rotations = $2(2\pi)$ radians = 4π radians

So $\omega = \frac{4\pi}{1}\ \frac{\text{rad}}{\text{min}} = 4\pi\ \frac{1}{\text{min}}$

Hence

$$v = r\omega = 5\text{ in}\left(4\pi\ \frac{1}{\text{min}}\right) = 20\pi\ \frac{\text{in}}{\text{min}}$$

Distance = rate times time

$$s = tv$$

$$30\text{ sec} = \frac{1}{2}\text{ min}$$

$$s = \left(\frac{1}{2}\text{ min}\right)\cdot\left(20\pi\ \frac{\text{in}}{\text{min}}\right) = 10\pi\text{ in}$$

Question

A wheel with a 30 cm radius rotates at a rate of 3 radians/sec. What is the linear speed of a point on its rim, in cm per second?

(a) $\frac{1}{10}$ cm/sec

(b) 10 cm/sec

(c) 33 cm/sec

(d) 90 cm/sec

(e) can't be determined without more information

$v = r\omega = 30 \text{ cm} \cdot 3 \frac{1}{\text{sec}} = 90 \frac{\text{cm}}{\text{sec}}$

$r = 30$ cm

$\omega = 3 \frac{1}{\text{sec}}$

Notes on Units

Remember that the formulas for

arclength, sector area, angular speed, & linear speed

are for an angle in **radians**. An angle in degrees must be converted to radians before applying any of these formulas.

Radians as *units*: We mentioned that radian measure is not a unit in the traditional sense. This is clear from the relationship

$$s = r\theta.$$

The lengths s and r would have the same units making

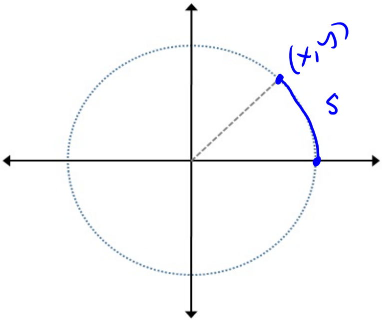
$$\theta = \frac{s}{r} \quad \text{unitless.}$$

Note then that the units for angular speed are *per time*. For example, x per second, or y per hour.

Section 6.5: Trigonometric Functions of a Real Variable and Their Graphs

Let s be **any real number.** Then we can equate an arc of the unit circle with s and consider the point (x, y) determined by s. Since the radius $r = 1$, the angle $\theta = s$. Hence

$$\sin s = y, \quad \cos s = x, \quad \text{and} \quad \tan s = \frac{y}{x}, \text{ when } x \neq 0.$$



Properties of Sine and Cosine

We can deduce some properties from the unit circle interpretation. One property is **periodicity**.

Definition: A function f is said to be **periodic** if there exists a positive constant p such that

$$f(x+p) = f(x)$$

for every x in the domain of f. The smallest such number p is called the **fundamental period** of the function f.

Recall that $f(x+p)$ corresponds to a horizontal shift—p units to the left for $p > 0$. Since $f(x+p) = f(x)$, the shifted graph must be indistinguishable from the unshifted graph.

Graph of a Periodic Function

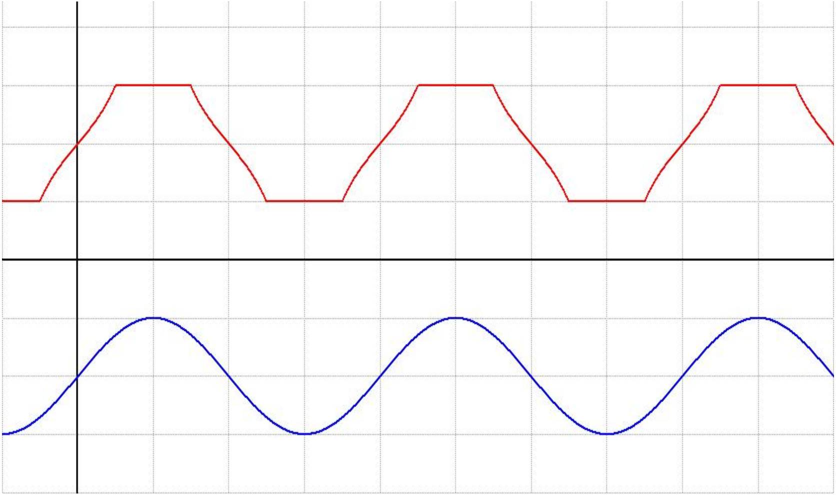


Figure: The profile of a periodic function repeats every p units.

Periodicity of Sine and Cosine

The sine and cosine function are periodic with fundamental period 2π. That is

$\cos(s+2\pi) = \cos s$ **and** $\sin(s+2\pi) = \sin s$ **for all real** s.