November 7 MATH 1113 sec. 52 Fall 2018

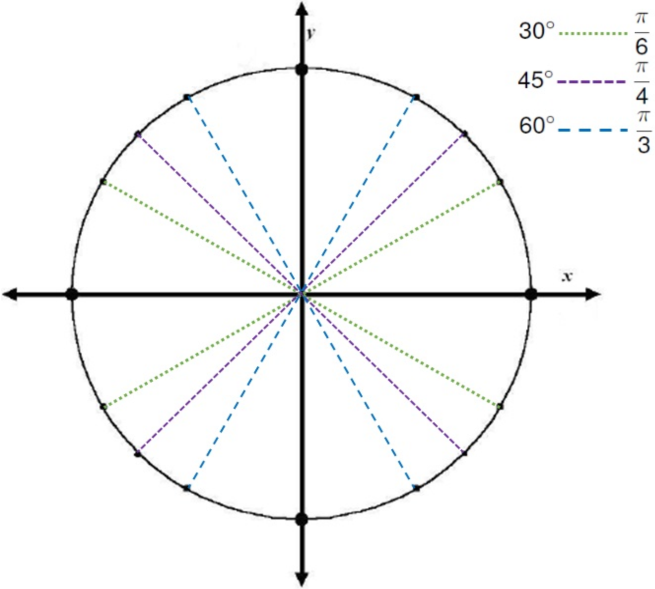
A quick sidebar on memorizing the unit circle

I expect you to *know* the following trigonometric values: (which can be deduced from the common triangles or unit circle)

θ°	0°	30°	45°	60°	90°
θ (rad.)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined

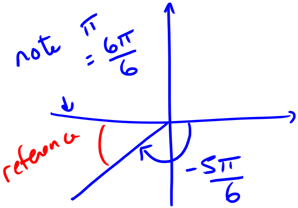
The rest of the *unit circle* can be obtained from these and from the signs of coordinates in the quadrants of the plane. (Remember the mnemonic "**A**ll **S**tudents **T**ake **C**alculus.")

The Unit Circle from Minimal Info



Example

Evaluate $\sin\left(-\frac{5\pi}{6}\right)$.



If $\theta = \frac{-5\pi}{6}$ then $\theta' = \frac{\pi}{6}$

$$\sin\left(\frac{-5\pi}{6}\right) = \pm \sin\left(\frac{\pi}{6}\right)$$

we need to find the correct sign

$$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$ (from memory)

Quad III $\Rightarrow$ Sine is negative

$$\sin\left(\frac{-5\pi}{6}\right) = -\frac{1}{2}$$

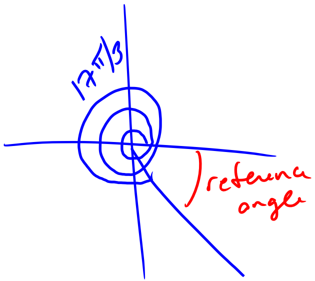
Example

Evaluate $\cot\left(\frac{17\pi}{3}\right)$.

$$\cot\left(\frac{17\pi}{3}\right) = \frac{1}{\tan\left(\frac{17\pi}{3}\right)}$$

3 rotations is $\frac{18\pi}{3}$

$\frac{17\pi}{3}$ is a bit less



If $\theta = \frac{17\pi}{3}$ then $\theta' = \frac{\pi}{3}$

$\tan\left(\frac{17\pi}{3}\right) = \pm \tan\left(\frac{\pi}{3}\right)$ we need the correct sign

$\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$ (from memory)

Quad IV $\Rightarrow$ tangent is negative

$\tan\left(\frac{17\pi}{3}\right) = -\sqrt{3}$

so $\cot\left(\frac{17\pi}{3}\right) = \frac{-1}{\sqrt{3}}$

Section 6.5: Trigonometric Functions of a Real Variable and Their Graphs

Here we consider the trigonometric functions as real valued functions of a real variable (real number inputs and outputs).

Recall: Let p be a positive real number. A function f is said to be periodic with period p provided

$$f(x+p)=f(x)$$

for each x in the domain of f.

The sine and cosine functions are periodic with fundamental period 2π. That is

$$\cos(s+2\pi)=\cos s \quad \text{and} \quad \sin(s+2\pi)=\sin s \quad \text{for all real } s.$$

Domain and Range and Amplitude

Every real number can be equated with a length of an arc (positive in the counter clockwise direction, negative in the clockwise). Hence

Domain: The domain of the sine function is **all real numbers**, and the domain of the cosine function is **all real numbers**.

Domain and Range and Amplitude

Every point (x, y) on the unit circle is such that $|x| \leq 1$ and $|y| \leq 1$. Hence

Range: The range of the sine function is $[-1, 1]$ and the range of the cosine function is also $[-1, 1]$.

The inequalities $|\sin x| \leq 1$ and $|\cos x| \leq 1$ are frequently used in mathematics.

Question

Suppose for some angle θ that $\tan\theta = \dfrac{5}{4}$.

Then $\sin\theta = 5$ and $\cos\theta = 4$.

(a) That must be true.

(b) That might be true.

(c) That cannot be true.

$-1 \le \sin\theta \le 1$

and

$-1 \le \cos\theta \le 1$

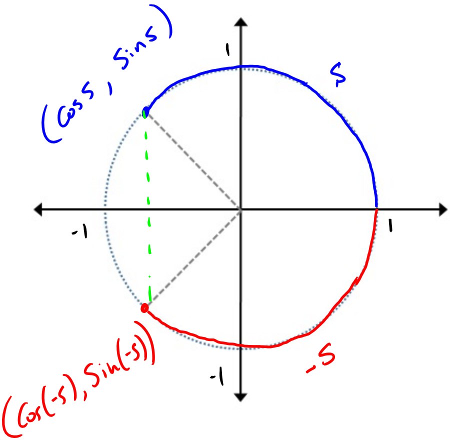
Domain and Range and Amplitude

Definition (Amplitude): The sine and cosine functions oscillate between their maximum and minimum values. Half of the distance between the maximum and minimum is called the **Amplitude**.

$$\boxed{\text{The amplitude of both } f(x) = \sin x \text{ and } g(x) = \cos x \text{ is } 1.}$$

Symmetry

Consider any real number s and its opposite $-s$.



The points have common x-value and y-values with opposite sign.

$$\cos s = \cos(-s)$$

$$\sin s = -\sin(-s)$$

$$\Downarrow$$

$$\sin(-s) = -\sin s$$

Symmetry of Sine and Cosine

Cosine is Even. The cosine function is an even function. Hence

$$\cos(-s) = \cos s \quad \text{for all real} \quad s.$$

Sine is Odd. The sine function is an odd function. Hence

$$\sin(-s) = -\sin s \quad \text{for all real} \quad s.$$

Graphs of Sine and Cosine

Due to periodicity and making use of symmetry, we can determine what the entire graphs of sine and cosine look like from the graphs on the interval $0 \le s < 2\pi$.

Here is an applet to plot one period of the functions $f(s) = \sin s$ and $f(s) = \cos s$.

GeoGebra Graph Applet: Sine and Cosine

Plot of $y = f(x) = \sin x$

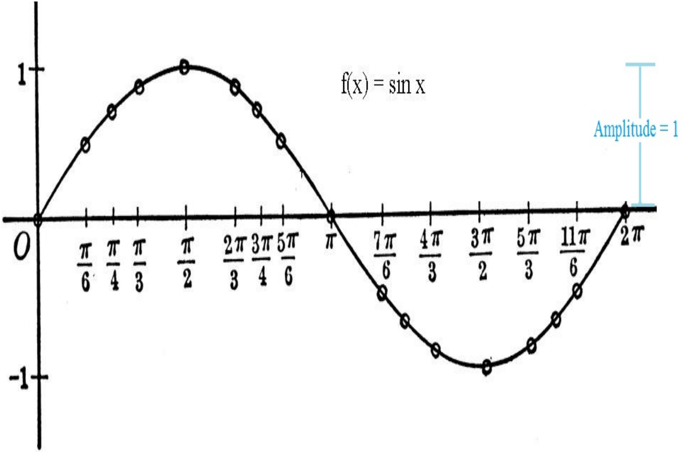


Figure: It's worth noting that major features (maximum, minimum, x-intercepts) divide the period into four equal parts.

Plot of $y = f(x) = \sin x$

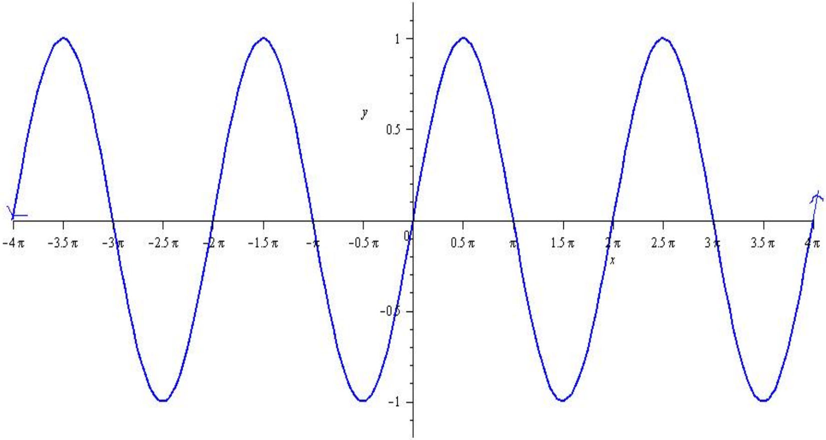


Figure: $y = \sin x$ over four full periods. Note the periodicity and the odd symmetry.

Plot of $y = f(x) = \cos x$

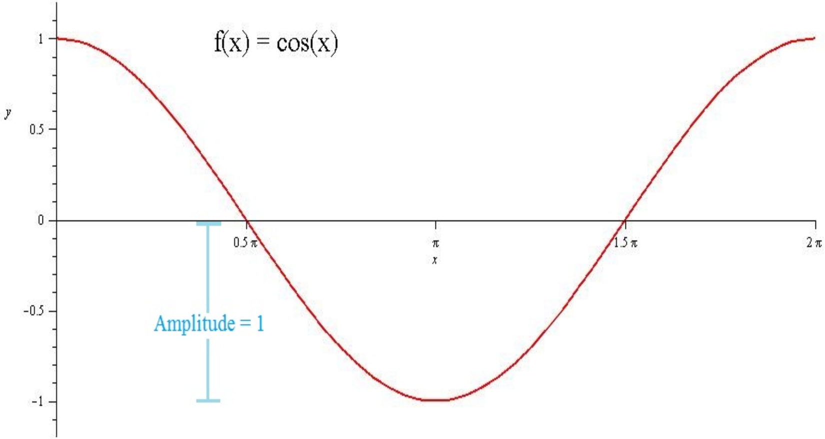


Figure: $y = \cos x$ over one full period.

Plot of $y = f(x) = \cos x$

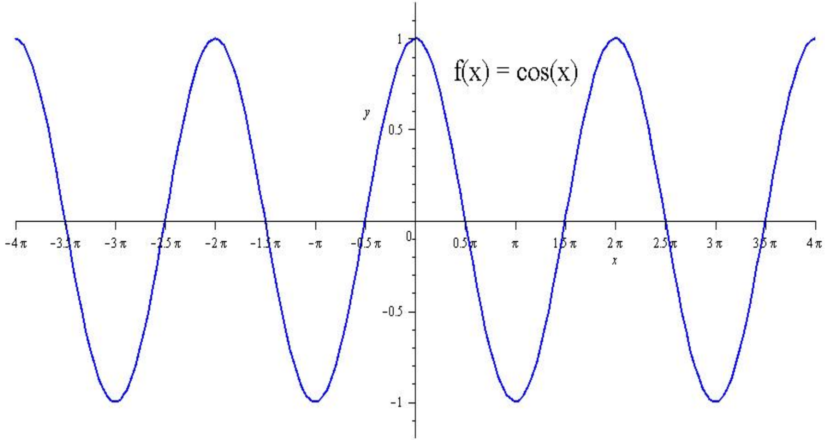


Figure: $y = \cos x$ over four full periods. Note the periodicity and the even symmetry

Plot of $f(x) = \cos x$ and $g(x) = \sin x$ Together

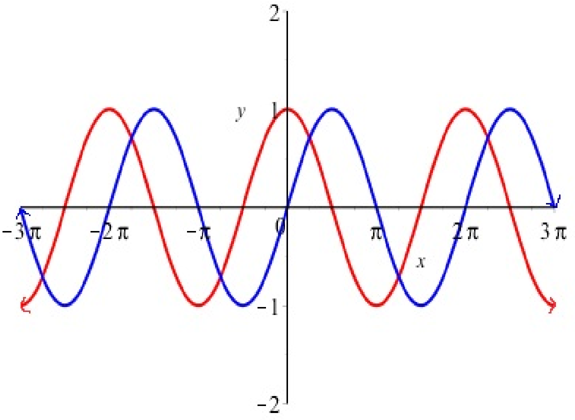


Figure: Note that the cofunction property $\cos(x) = \sin(\pi/2 - x)$ can be seen in the graphs.