November 7 Math 2306 sec. 56 Fall 2017

Section 16: Laplace Transforms of Derivatives and IVPs

A 1 kg mass is attached to a spring with spring constant 10 N/m. A dashpot induces damping numerically equation to 2 times the instantaneous velocity. A driving force of $f(t) = 18e^{-t}$ is imposed. If the mass starts at equilibruim from rest, determine the displacement for t > 0. Use the method of Laplace transforms.

$$mx'' + \beta x' + kx = f(k)$$
 $m = 1, \beta = 2, k = 10$

$$x'' + 2x' + 10x = 18e^{t} \qquad x(0) = 0, x'(0) = 0$$

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$$S^{2} \times (s) - S \times (0) - X'(0) + 2 \left(S \times (s) - X(0) \right) + 10 \times (s) = \frac{18}{S+1}$$

$$(s^2 + 2s + 10) X(s) = \frac{19}{s+1}$$

$$\chi(s) = \frac{(s+1)(s^2+2s+16)}{(s+1)(s^2+2s+16)}$$

Dischainant

S2+ 25+10

Partiel fractions

$$\frac{18}{(s_{11})(s^{2}+2s+10)} = \frac{A}{S+1} + \frac{Bs+C}{s^{2}+2s+10}$$

It's irreducible

4 D F 4 P F F F F F F F

$$|8 = A(s^2 + 2s + 10) + (Bs + c)(s + 1)$$

$$= A(s^2 + 2s + 10) + B(s^2 + s) + C(s + 1)$$

$$\overline{Os^2 + Os + 18} = (\overline{A+B}) s^2 + (\overline{SA+B+C}) s + 10A+C$$

$$A+B=0 \Rightarrow B=-A$$

$$2A+B+C=0 \Rightarrow 2A-A+C=0$$

A+C=0

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$$\chi(s) = \frac{2}{s+1} - \frac{2s+2}{s^2+2s+10}$$

Complete the square

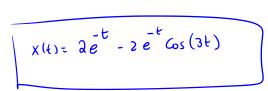
$$s^2 + 2s + 10 = s^2 + 2s + 1 + 9$$

= $(s+1)^2 + 9$

$$X(s) = \frac{2}{S+1} - 2 \frac{S+1}{(s+1)^2+3^2}$$

$$\frac{S+1}{(S+1)^2+3^2} = \frac{1}{S} \left\{ \cos(3+1) \right\}$$
Shifted

$$\times (\xi) = \mathcal{J} \left\{ \chi(s) \right\} = 2 \mathcal{J} \left\{ \frac{1}{5+1} \right\} - 2 \mathcal{J} \left\{ \frac{5+1}{(5+1)^2 + 3^2} \right\}$$



*
$$\tilde{\mathcal{Z}}'\{F(s-a)\}=e^{at}\tilde{\mathcal{Z}}'\{F(s)\}$$

$$\vec{y} \left\{ \frac{s+1}{(s+1)^2 + 3^2} \right\} = e^{-t} \vec{y} \left\{ \frac{s}{s^2 + 3^2} \right\}$$