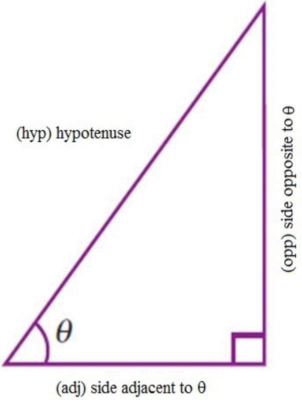
# October 19 MATH 1113 sec. 52 Fall 2018

## Sections 6.1 & 6.2: Trigonometric Functions of Acute Angles

We defined the six trigonometric values of an acute angle $\theta$ with reference to the triangle as labeled.



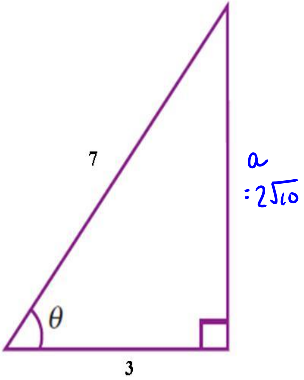

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

# Example

Determine the six trigonometric values of the acute angle $\theta$.




Calling the opposite side length $a$, $3^2 + a^2 = 7^2$

$$a^2 = 49 - 9 = 40$$

$$a = \sqrt{40} = 2\sqrt{10}$$

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{2\sqrt{10}}{7}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{3}{7}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{2\sqrt{10}}{3}$$

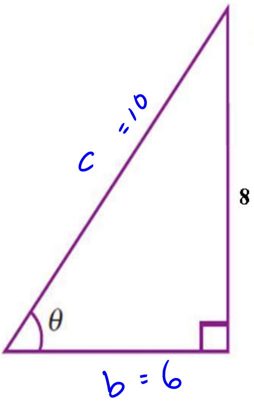
$$\csc\theta = \frac{1}{\sin\theta} = \frac{7}{2\sqrt{10}}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{7}{3}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{3}{2\sqrt{10}}$$

# Example

Determine the six trigonometric values of the acute angle $\theta$.




$$\sin\theta = \frac{4}{5} = \frac{opp}{hyp} = \frac{8}{c}$$

$$c = 10$$

$$b^2 + 8^2 = 10^2 \Rightarrow b^2 = 10^2 - 8^2$$

$$b = 6$$

$$\cos\theta = \frac{adj}{hyp} = \frac{6}{10} = \frac{3}{5}$$

$$\tan\theta = \frac{opp}{hyp} = \frac{8}{6} = \frac{4}{3}$$

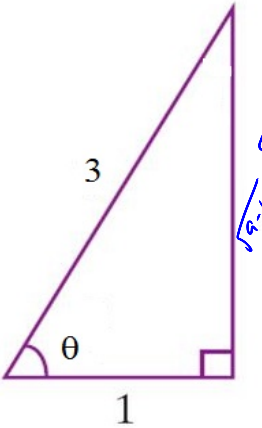
$$\csc\theta = \frac{1}{\sin\theta} = \frac{5}{4}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{3}{4}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{5}{3}$$

# Question

For the angle $\theta$ shown, which statement is correct?




$\sqrt{9-1} = \sqrt{8}$

(a) $\sin\theta = \frac{\sqrt{8}}{3}$ and $\cos\theta = \frac{1}{3}$

(b) $\sin\theta = \frac{1}{3}$ and $\cos\theta = \frac{\sqrt{2}}{3}$

(c) $\tan\theta = \frac{1}{3}$ and $\sin\theta = \frac{\sqrt{2}}{3}$

(d) $\tan\theta = \sqrt{2}$ and $\cot\theta = \frac{1}{\sqrt{2}}$

# Some Key Trigonometric Values

Use the triangles to determine the six trigonometric values of the angles $30^\circ, 45^\circ$, and $60^\circ$.

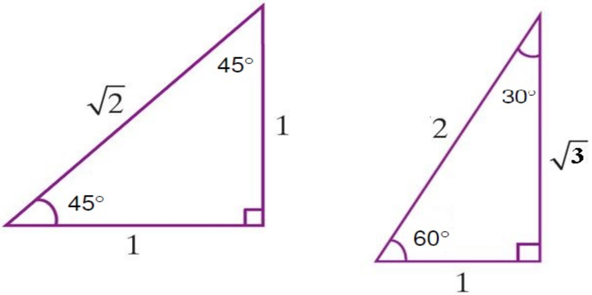



Figure: An isosceles right triangle of leg length 1 (left), and half of an equilateral triangle of side length 2 (right).

$\sin 45^\circ = \frac{1}{\sqrt{2}}$ $\sin 30^\circ = \frac{1}{2}$ $\sin 60^\circ = \frac{\sqrt{3}}{2}$

$\cos 45^\circ = \frac{1}{\sqrt{2}}$ $\cos 30^\circ = \frac{\sqrt{3}}{2}$ $\cos 60^\circ = \frac{1}{2}$

$\tan 45^\circ = 1$ $\tan 30^\circ = \frac{1}{\sqrt{3}}$ $\tan 60^\circ = \sqrt{3}$

$\csc 45^\circ = \sqrt{2}$ $\csc 30^\circ = 2$ $\csc 60^\circ = \frac{2}{\sqrt{3}}$

$\sec 45^\circ = \sqrt{2}$ $\sec 30^\circ = \frac{2}{\sqrt{3}}$ $\sec 60^\circ = 2$

$\cot 45^\circ = 1$ $\cot 30^\circ = \sqrt{3}$ $\cot 60^\circ = \frac{1}{\sqrt{3}}$

# Commit To Memory

It is to our advantage to remember the following:

$$\sin 30^\circ = \frac{1}{2}, \qquad \sin 45^\circ = \frac{1}{\sqrt{2}}, \qquad \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2}, \qquad \cos 45^\circ = \frac{1}{\sqrt{2}}, \qquad \cos 60^\circ = \frac{1}{2}$$

$$\tan 30^\circ = \frac{1}{\sqrt{3}}, \qquad \tan 45^\circ = 1, \qquad \tan 60^\circ = \sqrt{3}$$

We'll use these to find some other trigonometric values. Still others will require a calculator.

# Calculator


Figure: Any scientific calculator will have built in functions for sine, cosine and tangent. (TI-84 shown)

# Using a Calculator

Evaluate the following using a calculator. Round answers to three decimal places.

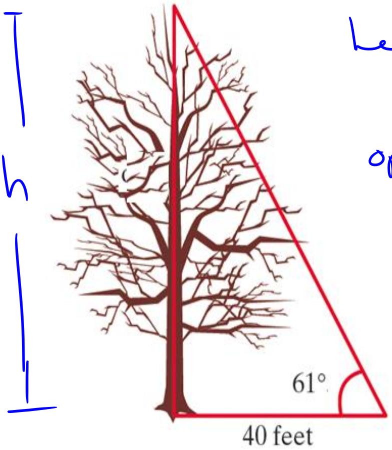
$\sin 16^\circ =$ 0.276

$\sec 78.3^\circ =$ $\dfrac{1}{\cos(78.3^\circ)} = 4.931$

$\tan(65.4^\circ) =$ 2.184

# Application Example

Before cutting down a dead tree, you wish to determine its height. From a horizontal distance of 40 ft, you measure the angle of elevation from the ground to the top of the tree to be $61^\circ$. Determine the tree height to the nearest $100^{th}$ of a foot.




Let $h$ be the tree height. Using the opposite and adjacent sides of the $61^\circ$ angle

$$\tan 61^\circ = \frac{h}{40\text{ ft}}$$

$$\cot 61^\circ = \frac{40\text{ft}}{h}$$

Using the 1st relation

$$h = (40\text{ ft})\tan 61^\circ$$

$$\approx 72.16 \text{ ft}$$