Oct. 19 Math 1190 sec. 51 Fall 2016

Section 4.3: The Mean Value Theorem

Rolle's Theorem: Let f be a function that is

i continuous on the closed interval $[a, b]$,

ii differentiable on the open interval (a, b), and

iii such that $f(a) = f(b)$.

Then there exists a number c in (a, b) such that $f'(c) = 0$.

The Mean Value Theorem (MVT) is arguably the most significant theorem in calculus. This is even accounting for a theorem we'll discuss later called the *Fundamental Theorem of Calculus*.

Rolle's Theorem

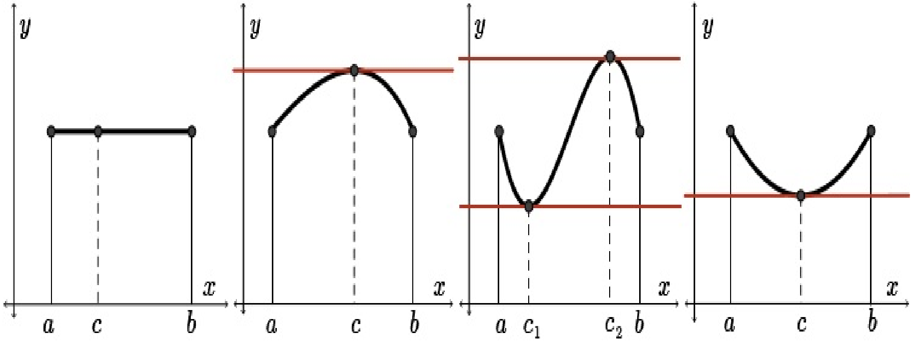
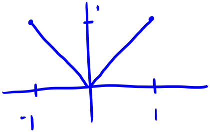


Figure: The theorem illustrated: How to find *c* is not indicated, but its existence is guaranteed.

Question

Let $f(x) = |x|$. Note that f is continuous on $[-1, 1]$ and $f(-1) = f(1) = 1$. However, **there is no** c **value in** $(-1, 1)$ **such that** $f'(c) = 0$. Which of the following is true?



(a) This contradicts Rolle's theorem.

(b) f is not really continuous on $[-1, 1]$, so Rolle's theorem doesn't apply.

(c) f is not differentiable on $(-1, 1)$, so Rolle's theorem doesn't apply.

The Mean Value Theorem

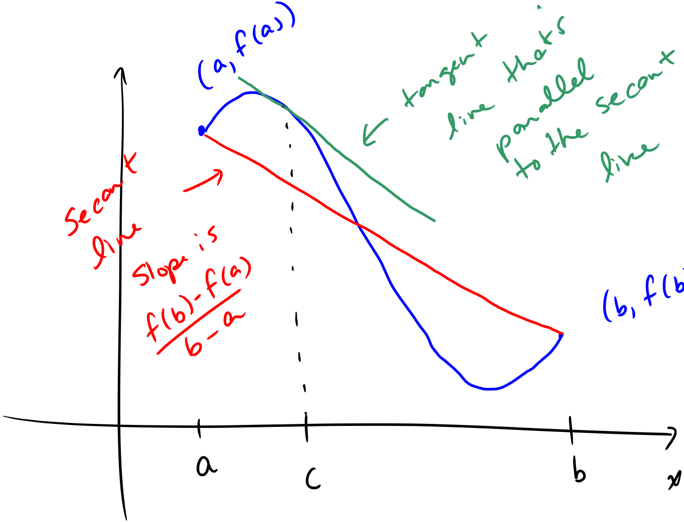
Theorem: Suppose f is a function that satisfies

i f is continuous on the closed interval $[a, b]$, and
ii f is differentiable on the open interval (a, b).

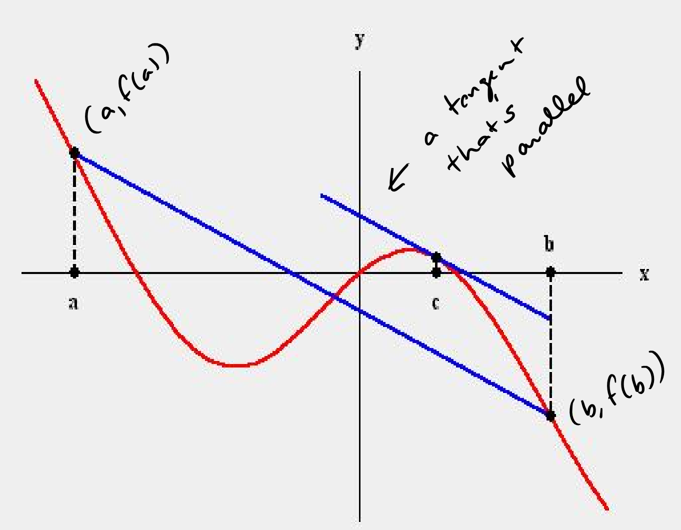
Then there exists a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}, \quad \text{equivalently} \quad f(b) - f(a) = f'(c)(b - a).$$

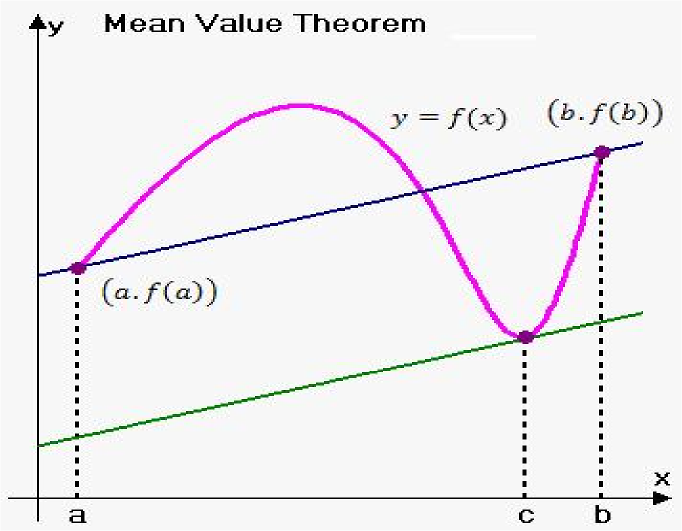
slope of the secant line connecting $(a, f(a))$ and $(b, f(b))$



(a, f(a))
tangent line that's parallel to the secant line
Secant line
Slope is $\frac{f(b)-f(a)}{b-a}$
(b, f(b))
a
c
b
x



Figure



Figure

Figure: Celebration of the MVT in Beijing.