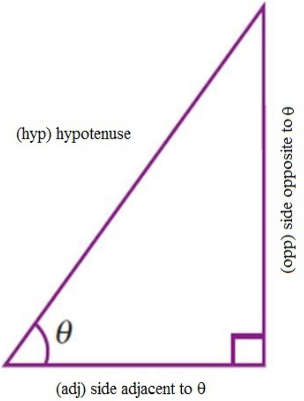
October 22 MATH 1113 sec. 51 Fall 2018

Sections 6.1 & 6.2: Trigonometric Functions of Acute Angles

We defined the six trigonometric values of an acute angle θ with reference to the triangle as labeled.



$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}}$$

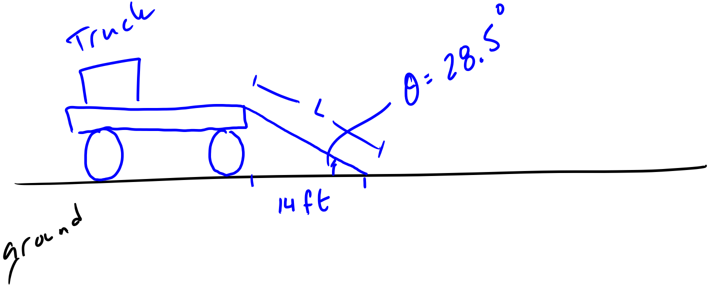
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

Application Example

The ramp of truck for moving touches the ground 14 feet from the end of the truck. If the ramp makes an angle of 28.5° with the ground, what is the length of the ramp?

Let L be the length of the ramp



Question

The ramp of truck for moving touches the ground 14 feet from the end of the truck. If the ramp makes an angle of 28.5° with the ground, what is the length of the ramp?

The length L of the ramp can be determined from the equation

(a) $\dfrac{L}{14} = \csc(28.5^\circ)$

(b) $\dfrac{L}{14} = \tan(28.5^\circ)$

(c) $\dfrac{14}{L} = \cos(28.5^\circ)$

(d) $\dfrac{14}{L} = \cot(28.5^\circ)$

L is hyp

14 ft is adj

$\dfrac{14}{L} = \dfrac{\text{adj}}{\text{hyp}} = \cos(28.5^\circ)$

Complementary Angles and Cofunction Identities

The two acute angles in a right triangle must sum to 90°. Two acute angles whose measures sum to 90° are called **complementary angles**. Given an acute angle θ its **complement** is the angle $90^\circ - \theta$.

Example Find the complementary angle of 27°.

The complement is

$$90^\circ - 27^\circ = 63^\circ$$

Cofunction Identities

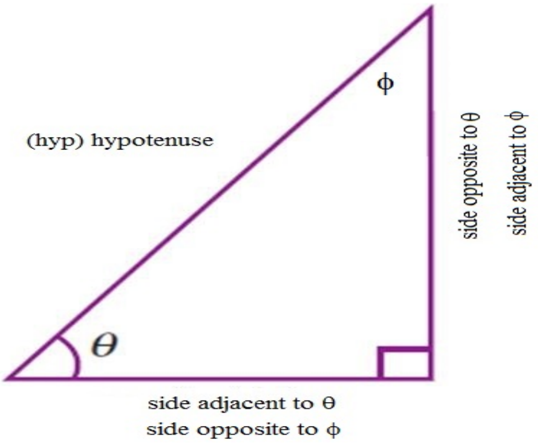


Figure: Note that for complementary angles θ and ϕ, the role of the legs (opposite versus adjacent) are interchanged.

Cofunction Identities

For any acute angle θ

$$\sin\theta = \cos(90^\circ - \theta) \qquad \cos\theta = \sin(90^\circ - \theta)$$

$$\tan\theta = \cot(90^\circ - \theta) \qquad \cot\theta = \tan(90^\circ - \theta)$$

$$\sec\theta = \csc(90^\circ - \theta) \qquad \csc\theta = \sec(90^\circ - \theta)$$

These equations define what are called **cofunction identities.**

Question

The value of $\sin(13^\circ)$ is equivalent to

(a) $\csc(77^\circ)$

(b) $\sin(77^\circ)$

(c) $\sec(77^\circ)$

(d) $\cos(77^\circ)$

The complement is

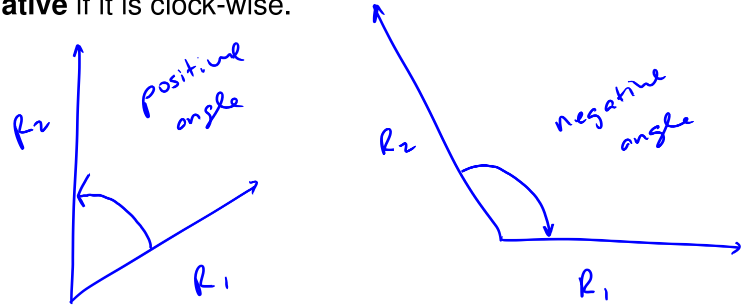
$90^\circ - 13^\circ = 77^\circ$

The cosine is the cofunction for the sine

Section 6.3: Angles, Rotations, and Angle Measures

We define an angle by a pair of rays (say R_1 and R_2) that share a common origin. We can indicate direction for an angle by indicating one ray as the **initial ray** and the other as the **terminal ray**.

We then define an angle as being **positive** if it is counter clock-wise and **negative** if it is clock-wise.



Angles in Standard Position

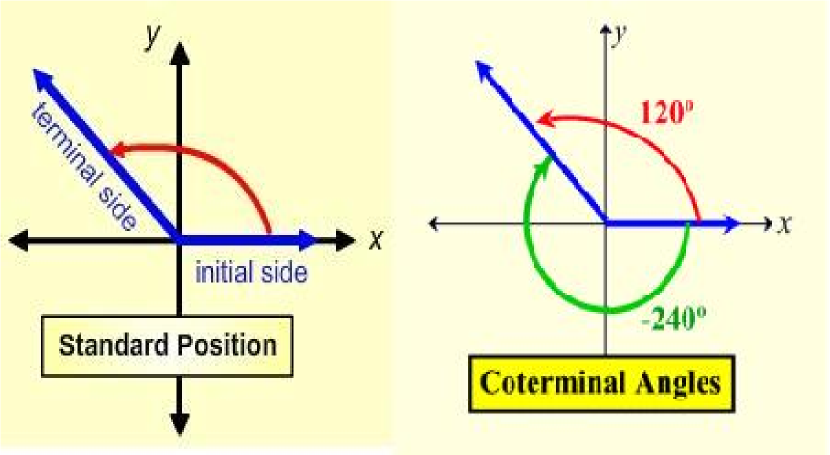


Figure: An angle in STANDARD POSITON has the $+x$-axis as its initial side. More than one angle may have the same terminal side. These are called *co-terminal*.

Degree Measure

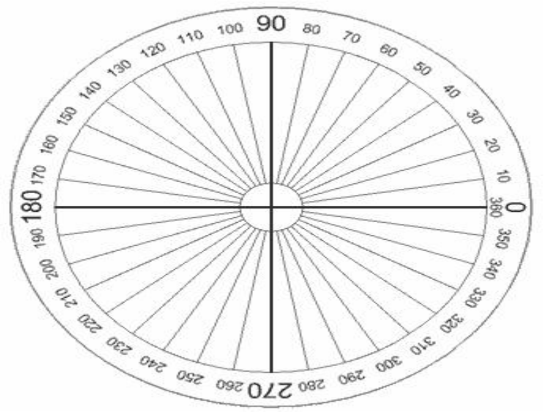


Figure: We can asign a measure to the angle between an initial and terminal side. **Degree** measure is obtained by dividing one full rotation into 360 equal parts.

Coterminal Angles

Since we can measure clockwise (neg.) or counter clockwise (pos.), and can allow for full rotations, angles in standard position may be coterminal but of different measure.

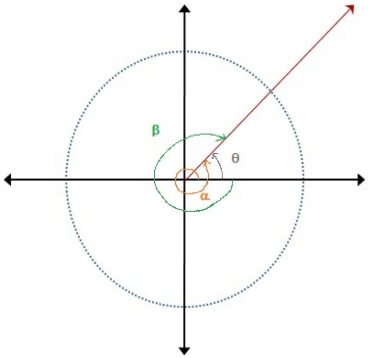
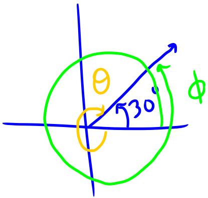


Figure: The three angles θ, α, and β have different measures but are coterminal. **Note: Coterminal angles differ by a multiple of** $360°$**.**

Example

Find two angles that are coterminal with 30°. Choose one that is negative, and one whose measure is greater than 360°.



Call the angles θ and ϕ

Let $\theta = 30^\circ - 360^\circ = -330^\circ$

Let $\phi = 30^\circ + 360^\circ = 390^\circ$

Question

Which of the following angles is coterminal with -45°?

(a) 45°

(b) 135°

(c) 225°

(d) 315°

Supplementary Angles

Recall that we called two actute angles whose measures sum to 90° **complementary** angles. We have a term for positive angles whose measures sum to 180°.

Definition: Two positive angles whose measures sum to 180° are called **supplementary** angles.

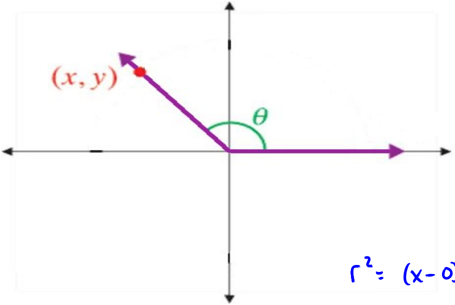
Example: Find the complementary and the supplementary angles for 38°.

The complement is $90^\circ - 38^\circ = 52^\circ$

The supplement is $180^\circ - 38^\circ = 142^\circ$

Trigonometric Functions of any Angle

We wish to extend the definitions of the six trigonometric functions to angles that are not necessarily acute. To start, consider an angle in standard position, and choose a point (x, y) on the terminal side.



Let r be the distance from $(0,0)$ to (x,y)

$r^2 = (x-0)^2 + (y-0)^2$

$r = \sqrt{x^2 + y^2}$

Trigonometric Function of any Angle

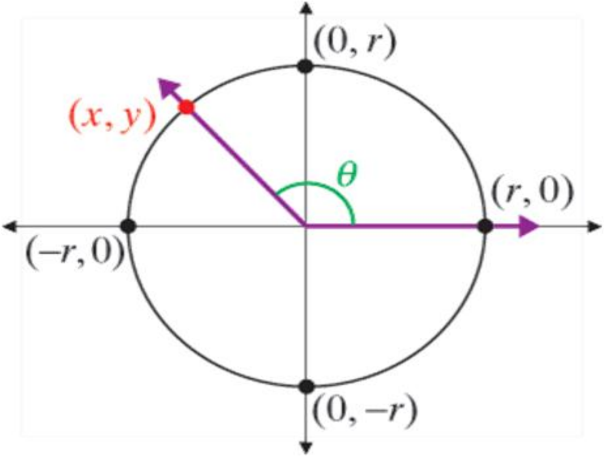


Figure: An angle in standard position determined by a point (x, y). Any such point lives on a circle in the plane centered at the origin having radius $r = \sqrt{x^2 + y^2}$

Trigonometric Function of any Angle

$$\sin\theta = \frac{y}{r}$$

$$\cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x} \quad (\text{for } x \neq 0)$$

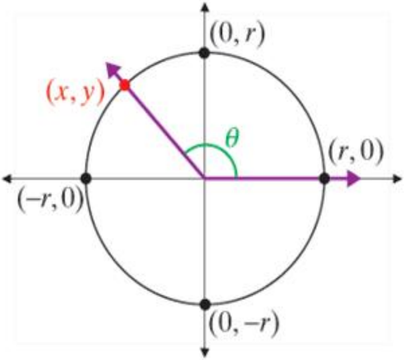


Figure: The definitions for the sine, cosine and tangent of any angle θ are given in terms of x, y, and r.

Trigonometric Function of any Angle

$$\csc\theta = \frac{r}{y} \quad (\text{for } y \neq 0)$$

$$\sec\theta = \frac{r}{x} \quad (\text{for } x \neq 0)$$

$$\cot\theta = \frac{x}{y} \quad (\text{for } y \neq 0)$$