October 24 MATH 1113 sec. 51 Fall 2018

Section 6.3: Angles, Rotations, and Angle Measures

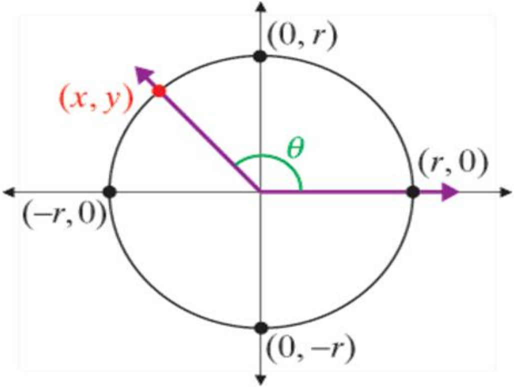


Figure: An angle in standard position determined by a point (x, y). Any such point lives on a circle in the plane centered at the origin having radius $r = \sqrt{x^2 + y^2}$

Trigonometric Function of any Angle

$$\sin\theta = \frac{y}{r}$$

$$\cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{y}{x} \quad (\text{for } x \neq 0)$$

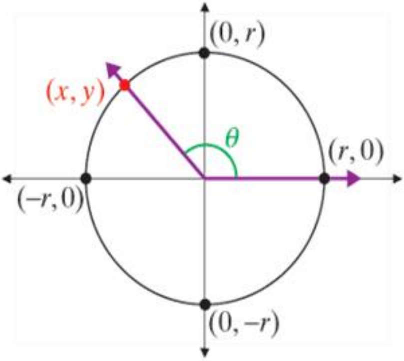


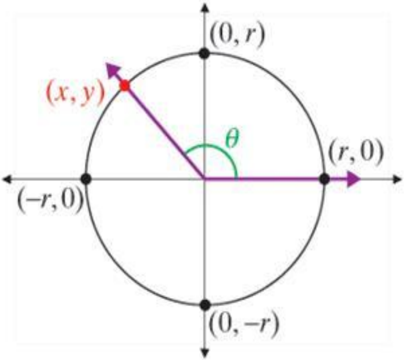
Figure: The definitions for the sine, cosine and tangent of any angle θ are given in terms of x, y, and r.

Trigonometric Function of any Angle

$$\csc\theta = \frac{r}{y} \quad (\text{for } y \neq 0)$$

$$\sec\theta = \frac{r}{x} \quad (\text{for } x \neq 0)$$

$$\cot\theta = \frac{x}{y} \quad (\text{for } y \neq 0)$$



Trigonometric Function of any Angle (Unit Circle Case)

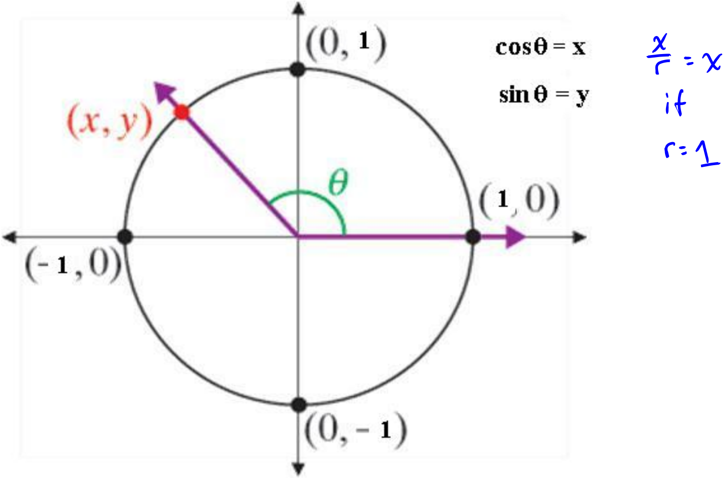


Figure: A point on the unit circle, $r = 1$, has coordinates $(x, y) = (\cos\theta, \sin\theta)$.

Reciprocal Identities

equation is always true when both sides are defined.

We have the first in a long list of **trigonometric identities**:

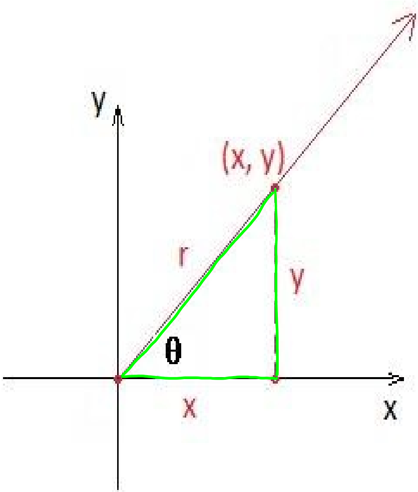
Reciprocal Identities: For any given θ for which both sides are defined

$$\csc\theta = \frac{1}{\sin\theta}, \quad \sec\theta = \frac{1}{\cos\theta}, \quad \& \quad \cot\theta = \frac{1}{\tan\theta}.$$

Equivalently

$$\sin\theta = \frac{1}{\csc\theta}, \quad \cos\theta = \frac{1}{\sec\theta}, \quad \& \quad \tan\theta = \frac{1}{\cot\theta}.$$

Comparison to Acute Angle Definitions



$r = \text{hyp}$ $\quad x = \text{adj}$ $\quad y = \text{opp}$

From before $\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$ same

Now $\sin\theta = \dfrac{y}{r}$

Similarly

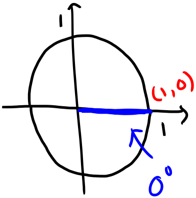
Old $\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$

New $\cos\theta = \dfrac{x}{r}$

Figure: Note that the acute angle definitions still hold.

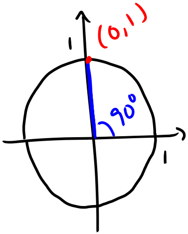
A Couple of Degenerate Triangles

Determine the sine, cosine, and tangent values of 0° and 90° as possible.



$\cos 0^\circ = 1 \qquad \sin 0^\circ = 0 \qquad (\cos\theta, \sin\theta) = (x, y)$

$\tan 0^\circ = \frac{0}{1} = 0$



$\cos 90^\circ = 0 \qquad \sin 90^\circ = 1$

$\tan 90^\circ = \text{"}\frac{1}{0}\text{"}$ that's undefined

Quadrantal Angles

The angles 0° and 90° both have the property that when put in standard position, the terminal side is concurrent with one of the coordinate axes.

Definition: Quadrantal angles are those angles that when put in standard position have terminal side concurrent with a coordinate axis. In addition to 0° and 90°, some quadrantal angles include

$$180^\circ, \quad 270^\circ, \quad -90^\circ, \quad \text{and} \quad 360^\circ$$

A Useful Table of Trigonometric Values

θ°	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undef.

Quadrants & Signs

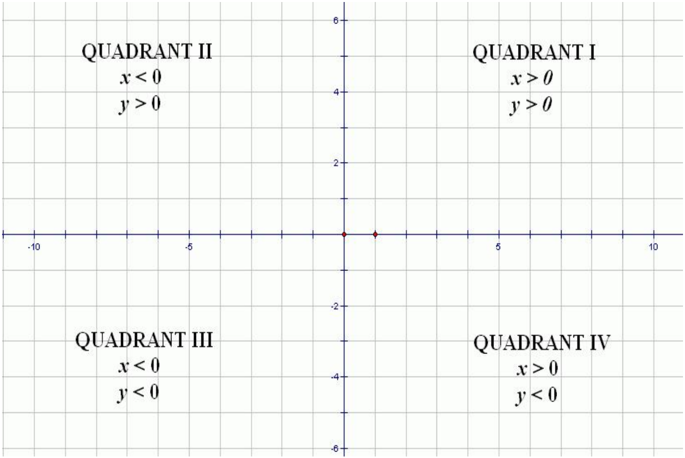
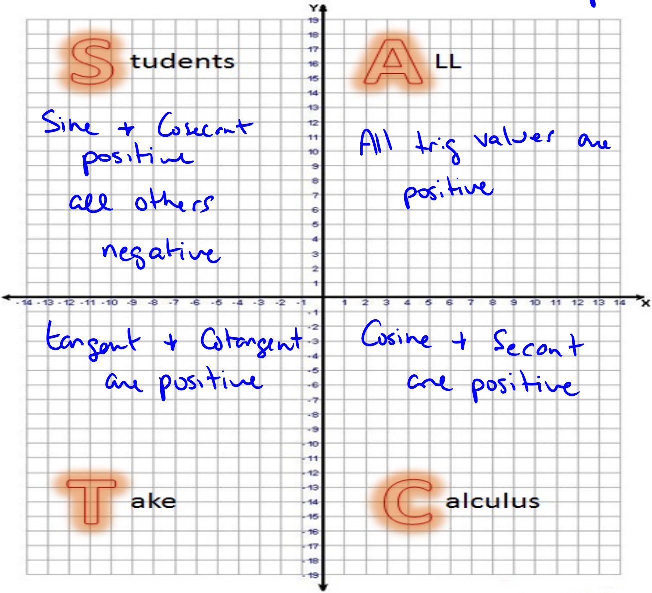


Figure: The trigonometric values for a general angle may be positive, negative, zero, or undefined. The signs are determined by the signs of the x and y values. **Note that $r > 0$ by definition.**

Quadrants & Signs of Trig Values

θ has terminal side in quadrant ...



Example

Determine which quadrant the terminal side of θ must be in if

(a) $\sin\theta > 0$ and $\tan\theta < 0$

$\Downarrow$ θ's terminal side in Quad I or II

$\Downarrow$ Quad II or IV

θ is a quad II angle

(b) $\sec\theta < 0$ and $\cot\theta > 0$

$\Downarrow$ Quad II or III

$\Downarrow$ Quad I or III

θ is in quad III

Question

for θ within one revolution

Suppose that $\sin\theta = -0.3420$ and $\cos\theta = -0.9397$. Which of the following must be true about θ?

quad III

(a) $0^\circ < \theta < 90^\circ$

(b) $90^\circ < \theta < 180^\circ$

(c) $180^\circ < \theta < 270^\circ$

(d) $270^\circ < \theta < 360^\circ$

(e) any of the above may be true, more information is needed to determine which is true

Reference Angles

Suppose we want to find the trig values for the angle θ shown. Note that the acute angle (pink) has terminal side through (x, y), and by symmetry the terminal side of θ passes through the point $(-x, y)$ (same y and opposite sign x).

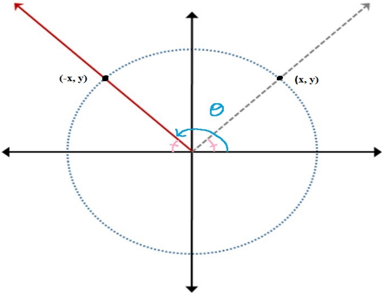


Figure: What is the connection between the trig values for θ and those for the acute angle in pink?

Reference Angles

Definition: Let θ be an angle in standard position. The **reference angle** θ' associated with θ is the angle of measure $0^\circ < \theta' < 90^\circ$ between the terminal side of θ and the *nearest* part of the *x*-axis.