October 2 Math 2306 sec 54 Fall 2015

Section 4.4: Method of Undetermined Coefficients

Using superposition as needed, begin with assumption:

$$y_p = y_{p_1} + \cdots + y_{p_k}$$

where y_{p_i} has the same **general form** as $g_i(x)$.

Case I: y_p as first written has no part that duplicates the complementary solution y_c. Then this first form will suffice.

Case II: y_p has a term y_{p_i} that duplicates a term in the complementary solution y_c. Multiply that term by x^n, where n is the smallest positive integer that eliminates the duplication.

Case II Examples

Solve the ODE

$$y'' - 2y' + y = -4e^x$$

Find y_c: solve $y'' - 2y' + y = 0$

$m^2 - 2m + 1 = 0 \Rightarrow (m-1)^2 = 0 \Rightarrow m = 1$ repeated

$y_1 = e^x, \quad y_2 = xe^x$

so $y_c = c_1 e^x + c_2 x e^x$

Find y_p: $g(x) = -4e^x$

$$y_p = \left(Ae^x\right)x^2 = \left(Axe^x\right)x = Ax^2e^x$$

part of y_c

So is this

This is not part of y_c

$$y_p = Ax^2e^x, \quad y_p' = 2Axe^x + Ax^2e^x, \quad y_p'' = 2Ae^x + 2Axe^x + 2Axe^x + Ax^2e^x$$

$$y_p'' - 2y_p' + y_p = -4e^x$$

$$2Ae^x + 4Axe^x + Ax^2e^x - 2\left(2Axe^x + Ax^2e^x\right) + Ax^2e^x = -4e^x$$

$$(A-2A+A)\,x^2e^x + (4A-4A)\,xe^x + 2Ae^x = -4e^x$$

$$2Ae^x = -4e^x$$

$$A = -2$$

So $y_p = -2x^2e^x$

The general solution is

$$y = c_1 e^x + c_2 x e^x - 2x^2 e^x.$$

Find the form of the particular soluition

$$y''' - y'' + y' - y = \cos x + x^4$$

$m^3 - m^2 + m - 1 = 0$ Find y_c

$m^2(m-1) + 1(m-1) = 0$

$(m-1)(m^2+1) = 0 \Longrightarrow$ $m-1=0$ or $m^2+1=0$

$m=1$ or $m = \pm i$

$= 0 \pm 1i$

$\alpha = 0, \beta = 1$

$y_1 = e^x$

$y_2 = \cos x$ $y_c = c_1 e^x + c_2 \cos x + c_3 \sin x$

$y_3 = \sin x$

Let $g_1(x) = \cos x$ $\qquad$ $g_2(x) = x^4$

part of y_c

$$y_{p_1} = (A\cos x + B\sin x)x$$

$$= Ax\cos x + Bx\sin x$$

this works

$$y_{p_2} = Cx^4 + Dx^3 + Ex^2 + Fx + G$$

this works

$$y_p = Ax\cos x + Bx\sin x + Cx^4 + Dx^3 + Ex^2 + Fx + G$$