October 31 MATH 1113 sec. 51 Fall 2018

Section 6.3: Angles, Rotations, and Angle Measures

Reference Angles Suppose we want to find the trig values for the angle θ shown. Note that the acute angle (pink) has terminal side through (x, y), and by symmetry the terminal side of θ passes through the point $(-x, y)$ (same y and opposite sign x).

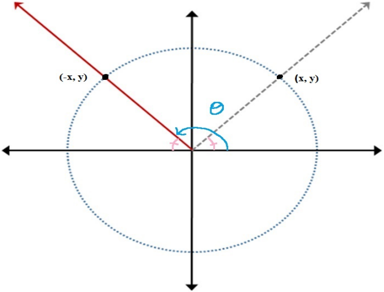
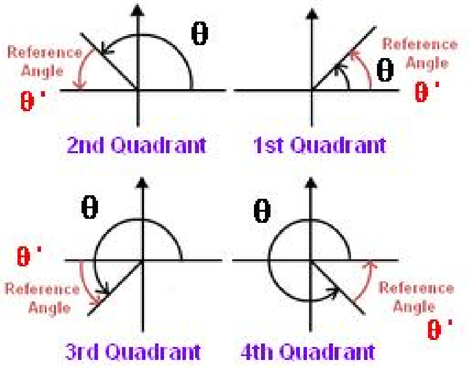


Figure: What is the connection between the trig values for θ and those for the acute angle in pink?

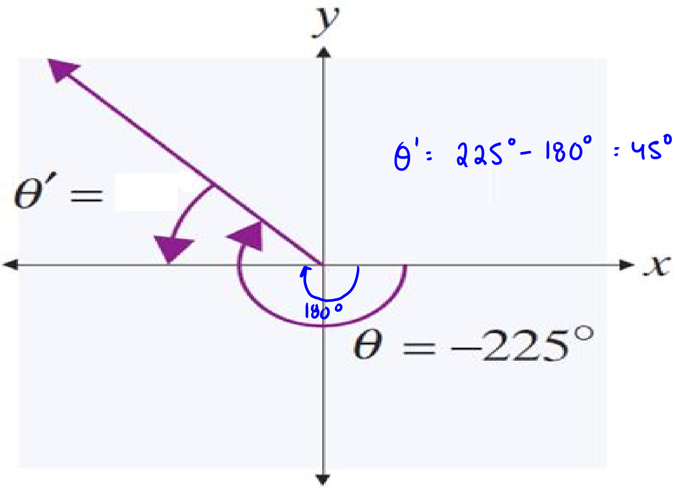
Reference Angles

Definition: Let θ be an angle in standard position. If θ is not a quadrantal angle, then the **reference angle** θ' associated with θ is the angle of measure $0^\circ < \theta' < 90^\circ$ between the terminal side of θ and the *nearest* part of the x-axis.



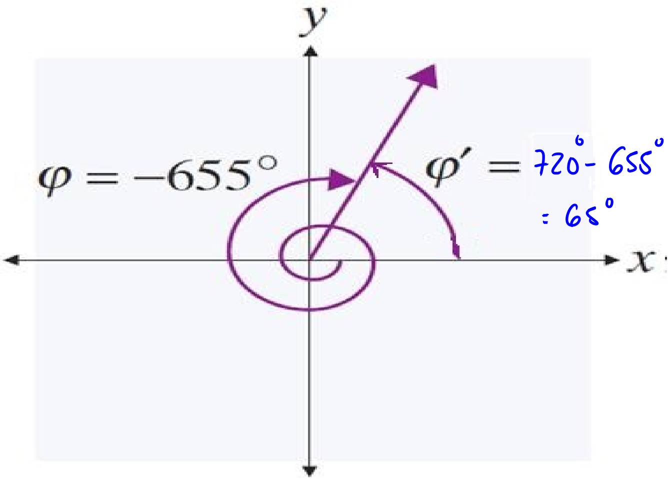
Example (a)

Determine the reference angle.



Example (b)

Determine the reference angle.



Question

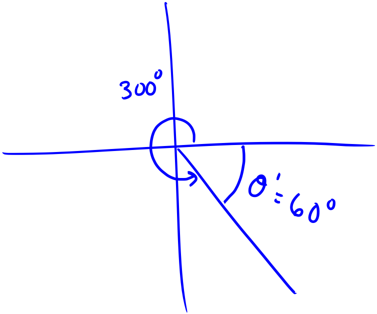
The reference angle for 300° is

(a) -60°

(b) 60°

(c) -30°

(d) 30°



Theorem on Reference Angles

Theorem: If θ' is the reference angle for the angle θ, then

$$\sin\theta' = |\sin\theta|, \quad \cos\theta' = |\cos\theta| \quad \& \quad \tan\theta' = |\tan\theta|.$$

Remark 1: The analogous relationships hold for the cosecant, secant, and cotangent.

Remark 2: This means that the trigonometric values for θ can differ at most by a sign (+ or -) from the values for θ'.

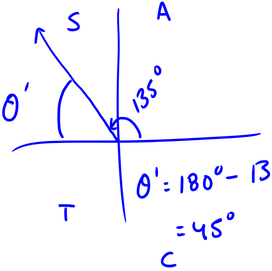
Example: Using Reference Angles

Find the exact value of

(a) $\sin(135^\circ)$

$\theta = 135^\circ$

$90^\circ < \theta < 180^\circ$



$\theta' = 180^\circ - 135^\circ$

$= 45^\circ$

$\sin(135^\circ) = \pm \sin(45^\circ)$

Choose correct sign

Quad II $\Rightarrow$ Sine is positive

$\sin(45^\circ) = \frac{\sqrt{2}}{2}$

so $\sin(135^\circ) = \frac{\sqrt{2}}{2}$

Example: Using Reference Angles

Find the exact value of

$\theta = 210^\circ$ $\quad 180^\circ < \theta < 270^\circ$

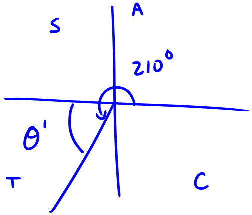
(b) $\cos(210^\circ)$

$\cos(210^\circ) = \pm \cos(30^\circ)$

Choose correct sign

$\cos\theta < 0$ in quad $\overline{\underline{\text{III}}}$

so $\cos(210^\circ) = \frac{-\sqrt{3}}{2}$



$\theta' = 210^\circ - 180^\circ = 30^\circ$

$\cos(30^\circ) = \frac{\sqrt{3}}{2}$

Question

Suppose θ is an angle such that $270^\circ < \theta < 360^\circ$ and its reference angle θ' satisfies

quadrant IV

$$\cos\theta' = \frac{2}{3}.$$

The secant of θ,

$$\sec\theta = \frac{1}{\cos\theta}$$

(a) $\sec\theta = \frac{3}{2}$, and I'm certain

(b) $\sec\theta = \frac{3}{2}$, but I'm not certain

and $\cos\theta > 0$ in quad IV

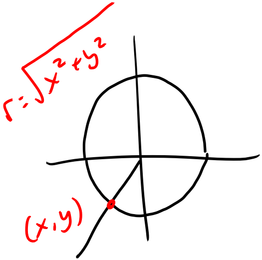
(c) $\sec\theta = -\frac{3}{2}$, and I'm certain

(d) $\sec\theta = -\frac{3}{2}$, but I'm not certain

More New Trigonometric Identities

Quotient Identities: For any given θ for which both sides are defined

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \quad \& \quad \cot\theta = \frac{\cos\theta}{\sin\theta}.$$



$\sin\theta = \frac{y}{r}$ and $\cos\theta = \frac{x}{r}$ $\quad \tan\theta = \frac{y}{x}$

so $\frac{\sin\theta}{\cos\theta} = \frac{y/r}{x/r} = \frac{y}{r}\cdot\frac{r}{x} = \frac{y}{x} = \tan\theta$

Example

Use the given information to determine the remaining trigonometric values of θ.

(a) $\sin\theta = \dfrac{1}{4}$ and $\cos\theta = -\dfrac{\sqrt{15}}{4}$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{4}{1} = 4$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{-4}{\sqrt{15}}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{1/4}{-\sqrt{15}/4} = \frac{1}{4}\cdot\frac{4}{-\sqrt{15}} = \frac{-1}{\sqrt{15}}$$

and

$$\cot\theta = \frac{1}{\tan\theta} = -\sqrt{15}$$

Question

Suppose we know that $\cos\theta = \dfrac{2}{\sqrt{13}}$ and $\cot\theta = -\dfrac{2}{3}$

Which of the following must be true?

(a) θ has terminal side in quadrant 4 ✓ $\cos\theta > 0$ I or IV $\cot\theta < 0$ II or IV

(b) $\sin\theta = -\dfrac{3}{\sqrt{13}}$ ✓ $\cot\theta = \dfrac{\cos\theta}{\sin\theta} \Rightarrow \sin\theta = \dfrac{\cos\theta}{\cot\theta}$

(c) for reference angle θ', $\tan\theta' = \dfrac{3}{2}$ ✓ $\tan\theta' = \left|\dfrac{1}{\cot\theta}\right|$

(d) All of the above are true.

(e) None of the above is true.