Oct 3 Math 2306 sec. 53 Fall 2018

Section 9: Method of Undetermined Coefficients

The context here is linear, constant coefficient, nonhomogeneous equations

$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_0 y = g(x)$$

where g comes from the restricted classes of functions

- polynomials,
- exponentials, 1/2

- sines and/or cosines,
- and products and sums of the above kinds of functions

Recall $y = y_c + y_p$, so we'll have to find both the complementary and the particular solutions!

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Motivating Example

Find a particular solution of the ODE

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be require

$$y_{p}^{*} - 4y_{p}^{*} + 4y_{p} = 8x + 1$$

$$y_{p}^{*} = A$$

$$y_{p}^{*} = 0$$

$$0 - 4(A) + 4(Ax + B) = 8x + 1$$

$$-4A + 4B = 8x + 1$$

$$4Ax + (-4A + 4B) = 8x + 1$$

$$4Ax + (-4A + 4B) = 8x + 1$$

$$4Ax + (-4A + 4B) = 8x + 1$$

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$$4Ax + (-4A + 4B) = 8x + 1$$

$$\begin{array}{cccc}
4A & = 8 \\
-4A & +4B & = 1
\end{array} \xrightarrow{A=Z} B = \frac{1}{4} (1 + 4A) = \frac{1}{4} (1 + 8) = \frac{9}{4} \end{array}$$

So the particular solution is
$$y_p = ax + \frac{q}{y}$$

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The Method: Assume y_p has the same **form** as g(x)

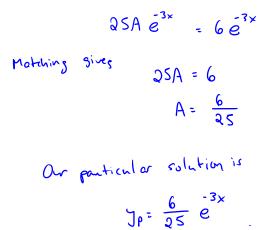
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$$y'' - 4y' + 4y = 6e^{-3x}$$

e'll assume that $y_{p} = A e^{-3x}$
substitute: $y_{p}' = -3A e^{-3x}$
 $y_{p}'' = 9A e^{-3x}$
 $QA e^{-3x} - 4(-3A e^{-3x}) + 4(Ae^{-3x}) = 6e^{-3x}$
 $e^{-3x} (9A + 12A + 4A) = 6e^{-3x}$

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Make the form general

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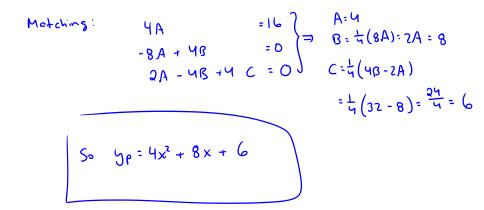
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Matching gives
$$YA = 16$$

-8A = 0 \Rightarrow A=4 and A=0
2A = 0 Not pussible

Substitute 2A - $4(2A \times + B) + 4(A \times + B \times + C) = 16 \times^{2}$

 $4A_{x}^{2} + (-8A + 4B)x + (2A - 4B + 4C) = 16x^{2} + 0x + 0$ $= 5x^{2} + 5x^{2} + 5x^{2} = 5x^{2} + 5x^{2} + 5x^{2} = 5x^{2} + 5x^{2} +$



General Form: sines and cosines

$$y''-y'=20\sin(2x)$$

If we assume that $y_p = A\sin(2x)$, taking two derivatives would lead to the equation

$$-4A\sin(2x) - 2A\cos(2x) = 20\sin(2x).$$

This would require (matching coefficients of sines and cosines)

$$-4A = 20$$
 and $-2A = 0$.

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This is impossible as it would require -5 = 0!

General Form: sines and cosines

We must think of our equation $y'' - y' = 20 \sin(2x)$ as

$$y'' - y' = 20\sin(2x) + 0\cos(2x).$$

The correct format for y_p is

$$y_p = A\sin(2x) + B\cos(2x).$$

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General Form: sines and cosines

$$y'' - y' = 20\sin(2x) + 0\cos(2x).$$

If $y_p = A\sin(2x) + B\cos(2x)$, then we get

$$y_p'' - y_p' = -4A\sin(2x) - 4B\cos(2x) - 2A\cos(2x) + 2B\sin(2x)$$

= 20 sin(2x) + 0 cos(2x).

We can match coefficients of sin(2x) and cos(2x) giving a **solvable** set of equations

-4A+2B=20 and $-2A-4B=0 \implies A=-4$, B=2.

The particular solution is

$$y_p = -4\sin(2x) + 2\cos(2x).$$

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Examples of Forms of y_p based on g (Trial Guesses)

(a) g(x) = 1 (or really any constant) Degree 0 polynomial $y_{p} = A$

(b) g(x) = x - 7 Pegree 1 poly.

yp = Ax+B

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Examples of Forms of y_p based on g (Trial Guesses)

(c)
$$g(x) = 5x^2$$
 Degree 2 poly.
 $y_p = A x^2 + B x + C$

(d)
$$g(x) = 3x^3 - 5$$
 Degre 3 polynomial
 $y_p = A x^3 + Bx^2 + Cx + D$

Examples of Forms of y_p based on g (Trial Guesses) (e) $g(x) = xe^{3x}$ Product of Degree 1 poly and e^{3x} $y_p = (A \times + B) e^{3x}$

(f)
$$g(x) = \cos(7x)$$
 Sum of Cosine + Sine 7x
 $y_p = A \cos(7x) + B \sin(7x)$

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