Oct 5 Math 2306 sec. 53 Fall 2018

Section 9: Method of Undetermined Coefficients

The context here is linear, constant coefficient, nonhomogeneous equations

$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_0 y = g(x)$$

where g comes from the restricted classes of functions

mv

- polynomials,
- exponentials, C
- ▶ sines and/or cosines, 5n(kx) or Cos(kx)
- and products and sums of the above kinds of functions

Recall $y = y_c + y_p$, so we'll have to find both the complementary and the particular solutions!

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Examples of Forms of y_p based on g (Trial Guesses)

(a) g(x) = 1 (or really any constant) $y_p = A$

(b) $g(x) = x - 7 y_p = Ax + B$

(c) $g(x) = 5x^2 y_p = Ax^2 + Bx + C$

(d) $g(x) = 3x^3 - 5 y_p = Ax^3 + Bx^2 + Cx + D$

(e) $g(x) = xe^{3x} y_{\rho} = (Ax + B)e^{3x}$

(f) $g(x) = \cos(7x) y_p = A\cos(7x) + B\sin(7x)$

Examples of Forms of y_p based on g (Trial Guesses)

(g) $g(x) = \sin(2x) - \cos(4x)$ $y_{p} = A S_{1}n(2x) + B C_{0s}(2x) + C S_{1}n(4x) + D C_{0s}(4x)$ (h) $g(x) = x^{2} \sin(3x)$ $\downarrow_{1}nes$ Sine and Cosine of 3x

 $y_{p^{2}}\left(A_{x^{2}+}B_{x+C}\right)S_{in}(3_{x}) + \left(D_{x^{2}+}E_{x}+F\right)C_{os}(3_{x})$

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Examples of Forms of y_p based on g (Trial Guesses)

(i)
$$g(x) = e^x \cos(2x)$$

 Lineer combo of e^x times since
 and cosine of $2x$.

yp: A & Cos(2x) + B & Sin(2x)

(j)
$$g(x) = xe^{-x}\sin(\pi x)$$

 1^{st} degree poly times e^{-x} times e^{-x}

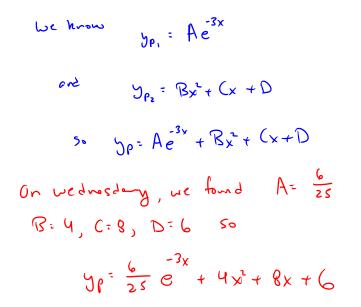
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The Superposition Principle

Example: Determine the correct form of the particular solution using the method of undetermined coefficients for the ODE

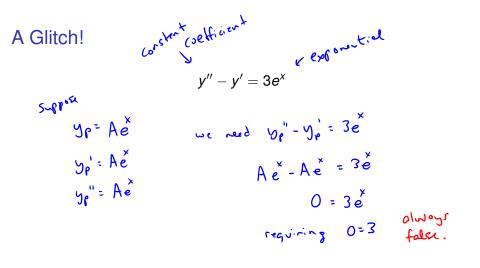
$$y'' - 4y' + 4y = 6e^{-3x} + 16x^{2}$$

By the principle of superposition, if
 $y_{P_{1}}$ solves $y'' - 4y' + 4y = 6e^{-3x}$
and $y_{P_{2}}$ solves $y'' - 4y' + 4y = 16x^{2}$
The solves $y'' - 4y' + 4y = 16x^{2}$



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The Characteristic equation is $m_{5} - w = 0$ m(m-1)=0two distinct real M=O or m=1 roots $y_1 = e^{0x} = 1$ $y_2 = e^{1x} = e^{0x}$ for yp solver the homogeneous Our guess $y_c = C_1 + C_2 e$ equation Spr Ae Puplicates yz well fix this by multiplying our "guess" by x (or x to some power) イロン イボン イヨン 一日 October 3, 2018

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$$y'' - y' = 3e^{x}$$
Let's try $y_{p} = A \times e^{x}$ substitute

$$y_{p}' = A \times e^{x} + Ae^{x}$$

$$y_{p}'' = A \times e^{x} + Ae^{x} + Ae^{x}$$

$$y_{p}'' = A \times e^{x} + Ae^{x} + Ae^{x}$$

$$= A \times e^{x} + 2Ae^{x}$$

$$y_{p}'' - y_{p}' = A \times e^{x} + 2Ae^{x} - (A \times e^{x} + Ae^{x}) = 3e^{x}$$
Collect like terms e^{x} and xe^{x}

$$xe^{x} (A - A) + e^{x} (2A - A) = 3e^{x}$$

A
$$\stackrel{\times}{e}$$
 = 3 $\stackrel{\times}{e}$
A=3
Thus y_{p} = 3x $\stackrel{\times}{e}$. The second solution
is $y_{=} c_{1} + c_{2} \stackrel{\times}{e} + 3x \stackrel{\times}{e}$

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We'll consider cases

$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_0 y = g_1(x) + g_2(x) + \cdots + g_i(x)$$

Using superposition as needed, begin with assumption:

$$y_p = y_{p_1} + \cdots + y_{p_k}$$

where y_{p_i} has the same **general form** as $g_i(x)$.

Case I: y_p as first written has no part that duplicates the complementary solution y_c . Then this first form will suffice.

Case II: y_{ρ} has a term y_{ρ_i} that duplicates a term in the complementary solution y_c . Multiply **that term** (only the y_{ρ_i} at issue) by x^n , where *n* is the smallest positive integer that eliminates the duplication.