# Oct 5 Math 2306 sec. 53 Fall 2018

#### Section 9: Method of Undetermined Coefficients

The context here is linear, constant coefficient, nonhomogeneous equations

$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_0 y = g(x)$$

where g comes from the restricted classes of functions

mv

- polynomials,
- exponentials, C
- ▶ sines and/or cosines, 5n(kx) or Cos(kx)
- and products and sums of the above kinds of functions

Recall  $y = y_c + y_p$ , so we'll have to find both the complementary and the particular solutions!

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### Examples of Forms of $y_p$ based on g (Trial Guesses)

(a) g(x) = 1 (or really any constant)  $y_p = A$ 

(b)  $g(x) = x - 7 y_p = Ax + B$ 

(c)  $g(x) = 5x^2 y_p = Ax^2 + Bx + C$ 

(d)  $g(x) = 3x^3 - 5 y_p = Ax^3 + Bx^2 + Cx + D$ 

(e)  $g(x) = xe^{3x} y_{\rho} = (Ax + B)e^{3x}$ 

(f)  $g(x) = \cos(7x) y_p = A\cos(7x) + B\sin(7x)$ 

Examples of Forms of  $y_p$  based on g (Trial Guesses)

(g)  $g(x) = \sin(2x) - \cos(4x)$   $y_{p} = A S_{1}n(2x) + B C_{0s}(2x) + C S_{1}n(4x) + D C_{0s}(4x)$ (h)  $g(x) = x^{2} \sin(3x)$   $\downarrow_{1}nes$  Sine and Cosine of 3x

 $y_{p^{2}}\left(A_{x^{2}+}B_{x+C}\right)S_{in}(3_{x}) + \left(D_{x^{2}+}E_{x}+F\right)C_{os}(3_{x})$ 

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# Examples of Forms of $y_p$ based on g (Trial Guesses)

(i) 
$$g(x) = e^x \cos(2x)$$
   
 Lineer combo of  $e^x$  times since   
 and cosine of  $2x$ .

yp: A & Cos(2x) + B & Sin(2x)

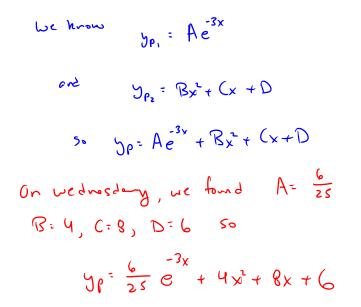
(j) 
$$g(x) = xe^{-x}\sin(\pi x)$$
   
 $1^{st}$  degree poly times  $e^{-x}$  times  $e^{-x}$ 

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# The Superposition Principle

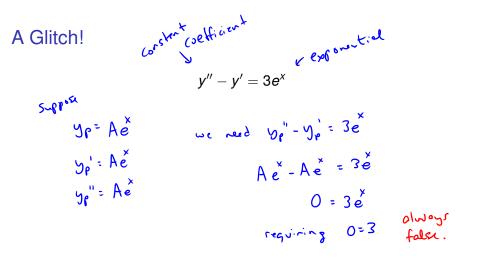
**Example:** Determine the correct form of the particular solution using the method of undetermined coefficients for the ODE

$$y'' - 4y' + 4y = 6e^{-3x} + 16x^{2}$$
  
By the principle of superposition, if  
 $y_{P_{1}}$  solves  $y'' - 4y' + 4y = 6e^{-3x}$   
and  $y_{P_{2}}$  solves  $y'' - 4y' + 4y = 16x^{2}$   
The solves  $y'' - 4y' + 4y = 16x^{2}$ 



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The Characteristic equation is  $m_{5} - w = 0$ m(m-1)=0two distinct real M=O or m=1 roots  $y_1 = e^{0x} = 1$   $y_2 = e^{1x} = e^{0x}$ for yp solver the homogeneous Our guess  $y_c = C_1 + C_2 e$ equation Spr Ae Puplicates yz well fix this by multiplying our "guess" by x (or x to some power) イロン イボン イヨン 一日 October 3, 2018

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$$y'' - y' = 3e^{x}$$
Let's try  $y_{p} = A \times e^{x}$  substitute  

$$y_{p}' = A \times e^{x} + Ae^{x}$$

$$y_{p}'' = A \times e^{x} + Ae^{x} + Ae^{x}$$

$$y_{p}'' = A \times e^{x} + Ae^{x} + Ae^{x}$$

$$= A \times e^{x} + 2Ae^{x}$$

$$y_{p}'' - y_{p}' = A \times e^{x} + 2Ae^{x} - (A \times e^{x} + Ae^{x}) = 3e^{x}$$
Collect like terms  $e^{x}$  and  $xe^{x}$ 

$$xe^{x} (A - A) + e^{x} (2A - A) = 3e^{x}$$

A
$$\stackrel{\times}{e}$$
 = 3  $\stackrel{\times}{e}$   
A=3  
Thus  $y_{p}$ = 3x $\stackrel{\times}{e}$ . The second solution  
is  $y_{=} c_{1} + c_{2} \stackrel{\times}{e} + 3x \stackrel{\times}{e}$ 

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### We'll consider cases

$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_0 y = g_1(x) + g_2(x) + \cdots + g_i(x)$$

Using superposition as needed, begin with assumption:

$$y_p = y_{p_1} + \cdots + y_{p_k}$$

where  $y_{p_i}$  has the same **general form** as  $g_i(x)$ .

**Case I:**  $y_p$  as first written has no part that duplicates the complementary solution  $y_c$ . Then this first form will suffice.

**Case II:**  $y_{\rho}$  has a term  $y_{\rho_i}$  that duplicates a term in the complementary solution  $y_c$ . Multiply **that term** (only the  $y_{\rho_i}$  at issue) by  $x^n$ , where *n* is the smallest positive integer that eliminates the duplication.